NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2106 PROBABILITY THEORY

Nov/Dec 2002 Time 3 hours

Candidates should attempt ALL questions in section A and THREE questions in section B. Marks will be allocated as indicated.

SECTION A [40 marks]

Candidates should attempt **ALL** questions being careful to number them A1 to A4.

A1. (a) State and explain the three axioms of probability.

(b) Prove that if the events A and B are independent, so are A' and B'. **[10 marks]**

A2. The cumulative distribution function of a continuous random variable X is given by

$$F(x) = \begin{cases} 0, x \leq 0 \\ 3x^2 - kx^3, 0 \leq x \leq 1 \\ 1, x \geq 1 \end{cases}$$

where $k \in \Re$

(a) Determine the value of k

(b) Hence, find

(i) $P(0 < X < \frac{1}{2})$

(ii) the probability density function of X

(iii) $E(X^3)$ **[10marks]**

A3. Let X be a random variable defined on a sample space S = {0, 1, 2,} with probability density function

$$P(X = x) = \begin{cases} p(1-p)^x, x \in S, 0 \leq p \leq 1 \\ 0, otherwise \end{cases}$$

Let $A = \{X \geq n\}$, $B = \{X > m\}$ and $C = \{X > m + n\}$

(i) Show that $P(X = x)$ is a discrete probability density function on S

(ii) Show that $P(C / B) = P(A)$ **[10marks]**

A4. (a) Prove that if $E_1, E_2, ..., E_n$ are independent events then

(b) Electric fans are shipped in batches of ten. Before a batch is accepted an inspector chooses three of these fans and inspects them. If none of the tested fans are defective, the batch is accepted. If one or more is found to be defective, the entire shipment is inspected. Suppose there are in fact two defective fans in the batch. What is the probability that 100% inspection is required? **[10marks]**

SECTION B [60marks]

Candidates may attempt **THREE** questions being careful to number them B5 through B8.

B5. (a) Prove that $M_{aX+b}(t) = e^{bt} M_X(at)$

(b) Suppose, e^{2t+4t^2} is the moment generating function of a random variable X. Let $Y = 2X + 1$.

(i) Find the distribution of X.

(ii) Find $P(-2 < X \leq 4)$

(iii) Find $P(|X| \leq 2)$

(iv) Find the moment generating function of Y

(v) Hence find the distribution of Y

[20marks]

B6. (a) State and prove Markov's inequality.

(b) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 50.

(i) Find the probability that this week's production will exceed 75?

(ii) If the variance of a week's production is known to be 25, find the probability that this week's production will be between 40 and 60?

(c) The probability that a resident of a new residential area in Bulawayo will install a 3-metre satellite was estimated and found to be 0.3. Find the probability that the 10th satellite dish to be installed in the area is the 5th 3-metre satellite dish. **[20marks]**

B7. (a) Let $E_1, E_2, ..., E_n$ be events.

(i) Prove that $P(E_1 \cap E_2) \geq P(E_1) + P(E_2) - 1$

(ii) Prove that $P(\bigcup_{i=1}^{n} E_i) = 1 - P(\bigcap_{i=1}^{n} E_i')$

(b) If the probability density function of X is given by

$$f(x) = \begin{cases} 2xe^{-x^2}, x > 0 \\ 0, otherwise \end{cases}$$

And let $Y = X^2$. Find

(i) The cumulative distribution function of Y

B8. (a) A small document is equally likely to be in any of three different folders. Let 2^{-i}, i=1, 2, 3 be the probability that you find the document in folder i, when the document is infact in folder i, assuming that the search is not thorough. Find

(i) The probability that the document is not found in the first folder

(ii) The probability that the document is found in folder 2 given that the document is not found in folder 1.

(b) A box contains 3 white, 3 blue and 4 black balls. Four balls are to be randomly chosen without replacement. Let X be the number of white balls and Y be the number of black balls. Find

(i) The joint probability distribution function of X and Y

(ii) P (X=x), P (Y=y), E (X) and E (Y)

(iii) The probability that the first two balls selected are white and the last two balls are black.

[20marks]

END OF EXAMINATION