NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2107 LINEAR PROGRAMMING

NOV/DEC 2002 Time: 3 hours

Answer **ALL** questions in Section A and **ANY FOUR** from section B

Section A [40 marks] Answer all questions from this section

A1. Briefly explain each of the following
- (i) Deterministic model
- (ii) Stochastic model
- (iii) Feasible solution
- (iv) Linear programming
- (v) Decision variable

[10 marks]

A2. (a) How do you transform the following linear constraint so that it can be used in linear programming

$$\frac{4x_1 + 5x_2 + 5x_3}{x_1 + x_2 + x_3} \geq 3$$

(b) State a condition necessary for a linear programming problem to have alternate optimal solutions.

(c) Why are artificial variables used in the simplex method? Will there ever be artificial variables in the optimal basis? Explain. [5 marks]

A3. A pharmaceutical firm has three salespersons that the firm wants to assign to three sales regions. Because of previously acquired contacts, the salespersons are able to cover the regions in different amounts of time. The amount of time in days required by each salesperson to cover each region is given in the table below:

	REGION		
Salesperson	A	B	C
Sue	10	15	9
Tapiwa	9	18	5
Ugenia	6	14	3

Which salesperson should be assigned to each region in order to minimize total time? Identify the optimal assignments and compute total minimum time. Use the Hungarian method to solve this problem. [10 marks]

A4. Solve graphically the following linear programming problem

Max $z = 4x_1 + 3x_2$

s. t.
$5x_1 - 4x_2 \le 40$
$x_2 \le 30$
$x_1 + x_2 \ge 0$

with $x_1, x_2 \ge 0$ [7 marks]

A5. A construction company has options to engage in six construction projects during the next two-year period. There is however only approximately $500 000 available for construction costs. The expected costs and expected net profits for the individual projects are listed in the table below.

Construction Project	Expected Net Profit ($000)	Expected Cost ($000)
A	180	125
B	120	90
C	100	60
D	140	120
E	105	75
F	200	150

Corporate policy places several additional restrictions on the project-selection decision

1. Exactly one of the projects A, B and C must be selected.
2. Exactly one of the projects B, C, D, E and F must be selected
3. At most one of the two projects E and F can be selected.
4. At most two of the projects A, B, C, D and E can be selected.

Formulate as a linear integer programming model. [8 marks]

SECTION B: [60 marks]

Answer **ANY FOUR** questions from this section

B6. A concrete company transports concrete from three plants to three construction sites. The supply capacities of the three plants, the demand requirements at the three sites and the transportation costs in ($000/ton) are given in the table below.

	To	construction	site	Supply
From Plant	1	2	3	(tons)
1	$8	$5	$6	120
2	$15	$10	$12	80
3	$3	$9	$10	80
Demand (tons)	150	70	60	280

(a) Illustrate this transportation problem as a network model
(b) Formulate as a linear programming problem
(c) Solve this problem using Vogel's Approximation Method
(d) Demand at construction site 3 has been decreased to 50 tons. In the light of this change solve using the least cost method. [15 marks]

B7 (a) Define integer linear programming.
(b) Given the all-integer linear program

Max $z = 2x_1 + 3x_2$

s. t.
$65x_1 + 91x_2 \leq 455$
$4x_1 + 40x_2 \leq 140$
$x_1 \leq 4$

with $x_1, x_2 \geq 0$ and integer

The optimal solution to the linear programming relaxation of this problem is $x_1 = 2.44$, $x_2 = 3.26$

(i) Find the optimal z-value of the relaxation optimal solution.
(ii) Round down to find a feasible integer solution and the corresponding z-value.
(iii) Develop the first two branches of the branch and bound method for solving this problem. You should also write down the two linear programming relaxation models at node 2 and node 3 given that your initial node is labeled node 1.

(c) The Ice-Cold Refrigerator Company can invest capital funds in a variety of projects that have varying capital requirements over the next four years. Faced with limited capital resources the company must select the most profitable projects and budgets for the capital expenditures. The estimated present values of the projects, the capital requirements and the available capital projections are shown in the table below:

	Capital Requirements (Expenditures ($000/year))				RETURNS
PROJECT	1	2	3	4	**($000/year)**
Plant Expansion	15	20	20	15	**90**
Warehouse Expansion	10	15	20	5	**40**
New Machinery	10	0	0	4	**10**
New product Research	15	10	10	10	**37**
Available Capital Funds	**40**	**50**	**40**	**35**	

Formulate an integer linear programming model which will determine the projects to be executed over the four-year horizon. [15 marks]

B8. The Animal Division of Kubatana Company produces stuffed hawks and doves. Under present market conditions Kubatana sells hawks at a profit of $4 and doves at a profit of $2. Hawk skins are tougher and take longer to make than dove skin; in particular, the machine that produces skins must be used for 15 seconds to make a hawk skin and 10 seconds to make a dove skin. On the other hand, hawks require less time to stuff than doves; in particular the stuffing machine must be used for 10 seconds to stuff a hawk and 15 seconds to stuff a dove. The daily availabilities of the skin machine and the stuffing machine are each 28 800 seconds. Hawks go through a final beak sharpening machine that can process at most 1680 hawks per day; dove beaks are not sharpened. A final constraint arises because Kubatana's managing director has a fondness for doves and has specified that at least 720 doves must be produced each day.

(a) Let x_H and x_D denote the daily production quantities of hawks and doves respectively. Formulate an LP for determining the product mix that maximizes total profit.

(b) Use the computer output attached in appendix A to analyze the effect of a change in a decision variable's objective-function coefficient or a structural constraint's right-hand side. In some parts, the computer output may provide insufficient information to allow you to answer, in which case you should state "Insufficient information!"

(i) Kubatana anticipates a change in market conditions that would increase the unit profit of doves from $2.00 to $2.50. If this increase occurs, what are the optimal values of the decision variables and what is the optimal objective value?

(ii) Kubatana anticipates a change in market conditions that would decrease the unit profit of hawks from $4.00 to $2.50. If this decrease occurs, what are

iii) If the availability of the skin machine increases from 28 800 to30 600 seconds, what is the optimal total profit?
(iv) If the availability of the stuffing machine increases from 28 800 to 30 600 seconds, what is the optimal total profit?
(v) If Kubatana's managing director were to reduce the minimum dove production from 720 to 600 how would Kubatana's total profit change.
(vi) If Kubatana's managing director were to reduce the minimum dove production from 720 to 350, how would Kubatana's total profit change? [15]

B9 (a) Define each of the following
(i) Degenerate solution
(ii) Alternate optimal solution
(iii) Unbounded solution
(iv) Infeasible solution
(b) Each of the following linear programming models falls under one of the situations described in (a). By using the graphical method show which linear programming models labeled 1 to 4 results in
(i) a degenerate solution
(ii) an unbounded solution
(iii) an infeasible solution
(iv) more than one solution

Linear Programming Model 1

Max $z = 3x_1 + 9x_2$
s. t. $x_1 + 2x_2 \le 4$
$x_1 + x_2 \le 8$
with $x_1, x_2 \ge 0$

Linear Programming Model 2

Max $z = 2x_1 + 4x_2$
s. t. $x_1 + x_2 \le 4$
$x_1 + 2x_2 \le 5$
with $x_1, x_2 \ge 0$

Linear Programming Model 3
Max $z = 2x_1 + x_2$
s. t. $x_1 - x_2 \le 10$
$2x_1 \le 40$
with $x_1, x_2 \ge 0$

Max $z = 3x_1 + 2x_2$

s. t. $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

with $x_1, x_2 \ge 0$

B10. Bevco manufactures an orange-flavoured floured soft drink called Oranj by combining orange soda and orange juice. Each gram of orange soda contains 0.5mg of sugar and 1mg of vitamin C. Each gram of orange juice contains 0.25mg of sugar and 3mg of vitamin C. It costs Bevco $2 to produce a gram of orange soda and $3 to produce a gram of orange juice. Bevco's marketing department has decided that each 10-gram bottle of Oranj must contain at least 20mg of vitamin C and at most 4mg of sugar. Bevco's marketing seeks to minimize cost.

(a) Formulate a linear programming model
(b) Solve using the two-phase method
(c) State one major disadvantage of using the big M method. [15 marks]

END OF QUESTION PAPER

APPENDIX A

TORA Optimization System - Version 2.0, Oct 1996

Date: Thu Oct 03 16:06:41 2002

*** OPTIMUM SOLUTION SUMMARY ***

Title: Kubatana Company
Final iteration No: 3
Objective value (max) = 7200.0000

Variable	Value	Obj Coeff	Obj Val Contrib
x1 xH	1440.0000	4.0000	5760.0000
x2 xD	720.0000	2.0000	1440.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (<)	28800.0000	0.0000-
2 (<)	28800.0000	3600.0000-
3 (<)	1680.0000	240.0000-
4 (>)	720.0000	0.0000+

*** SENSITIVITY ANALYSIS ***

Objective coefficients -- Single Changes:

Variable	Current Coeff	Min Coeff	Max Coeff	Reduced Cost
x1 xH	4.0000	3.0000	infinity	0.0000
x2 xD	2.0000	-infinity	2.6667	0.0000

Right-hand Side -- Single Changes:

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<)	28800.0000	7200.0020	32399.9998	0.2667
2 (<)	28800.0000	25200.0000	infinity	0.0000
3 (<)	1680.0000	1440.0000	infinity	0.0000
4 (>)	720.0000	360.0000	1152.0000	-0.6667

ight-hand Side Ranging -- Simultaneous Changes D:

Basic Var	Value/Feasibilty Condition			
x1 xH	1440.0000 +	0.0667 D1 +	-0.6667 D4 >= 0	
sx5	3600.0000 +	-0.6667 D1 +	1.0000 D2 +	-8.3333 D4
	>= 0			
sx6	240.0000 +	-0.0667 D1 +	1.0000 D3 +	0.6667 D4
	>= 0			
x2 xD	720.0000 +	1.0000 D4 >= 0		

End of Solution Summary