

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2107: Linear programming

August 2004 Supplementary Exams

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY FOUR questions from section B

Section A: Answer all questions in this section. [40]

- A1 Briefly explain the following
- (a) A constraint
 - (b) A linear programming model
 - (c) A feasible solution
 - (d) Slack variable
 - (e) Degeneracy

[10marks]

- A2 A pilot's diet requires that the food he/she eats come from one of the four basic food groups (chocolate cake, ice cream, soda and cheese cake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola and pine apple cheese. Each brownie costs \$50, each scoop of chocolate ice cream costs \$20, each bottle of cola costs \$30, and each pine of pineapple cheese costs \$80. Each day the pilot must ingest at least 500 calories, 6g of chocolate, 10g of sugar, and 8g of fat. The nutritional content per unit of each food is shown below.

	Calories	Chocolate	Sugar	Fat
Brownies	400	3	2	2
Chocolate ice cream	200	2	2	4
Cola	150	0	4	1
Pineapple cheese	500	0	4	5

Formulate a linear programming model that can be used to satisfy the pilot's daily nutritional requirements at minimum costs.

[7 marks]

A3 Use the simplex algorithm to solve the following problem

$$\begin{aligned} \text{Min } z &= 2x_1 - 3x_2 \\ \text{s.t. } \quad &x_1 + x_2 \leq 4 \\ &x_1 - x_2 \leq 6 \\ &x_1, x_2 \geq 0 \end{aligned}$$

[5marks]

A4 Solve the following transportation problem whose objective is to minimize total transportation costs.

		Destination				
		A	B	C	D	Supply
Source	1	2	3	5	6	5
	2	2	1	3	5	10
	3	3	8	4	6	15
Demand		12	8	4	6	

[6marks]

A5 Find the minimum cost of assignment of four jobs and four machines when the cost of assignment is given by the following table

	J1	J2	J3	J4
M1	1	4	6	3
M2	7	9	10	9
M3	4	5	11	7
M4	5	7	8	8

[5marks]

A6 Consider the all integer linear program given below

$$\begin{aligned} \text{Max } & 5x_1 + 8x_2 \\ & 6x_1 + 5x_2 \leq 30 \\ \text{s.t. } & 9x_1 + 4x_2 \leq 36 \\ & 1x_1 + 2x_2 \leq 10 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

- Graph the constraints for this problem using heavy dots to indicate all the feasible integer solutions.
- Find the optimal solution to the L.P Relaxation problem.
- Find the optimal integer solution to the above problem.

[3+3+2marks]

Section B: Answer FOUR Questions in this section. [60]

B7 Nyamz Sporting (Pvt) Ltd makes two different types of baseball gloves, a regular model and a catcher's model. The firm has 900 hrs of production time available in its cutting and sewing department, 300 hrs in its finishing department and 100 hrs available in its packaging and shipping department. The production time requirements and the profit contribution per glove are given in the table below

Model	Production time (hrs)			Profit per glove
	Cutting & sewing	Finishing	Packaging & shipping	
Regular	1	1/2	1/8	\$5
Catcher	3/2	1/3	1/4	\$8

The firm is interested in maximising the total profit contribution.

- Formulate the linear programming model for this problem [4 marks]
- By using the graphical method find the optimal solution i.e how many gloves of each model should the firm produce. [5 marks]
- What is the total profit contribution the firm can get from the optimal solution. [2 marks]
- How many hrs of production time will be scheduled in each department. [2 marks]
- What is the slack time in each department [2 marks]

- B8 The Mzenda Furniture Company manufactures desks, tables and chairs. The manufacture of each type of furniture requires mukwa and two types of skilled labor; finishing labor and carpentry labor. The amount of each resource needed to make each type of furniture is given below

Resource	Desk	Table	Chair
Mukwa	8m	6m	1m
Finishing hrs	4hrs	2hrs	1.5hrs
Carpentry hrs	2hrs	1.5hrs	0.5hrs

At present 48m of mukwa, 20 finishing hrs and 8 carpentry hrs are available. A desk sell for \$60, a table for \$30 and a chair for \$20. The company believes that demand for desks and chairs is unlimited, but at most 5 tables can be sold. The company wants to maximise total revenue.

- Formulate the Linear model for this problem
- Use the simplex method to find the optimal solution
- Give a brief comment on your results obtained in (b) above.

[4+8+3marks]

- B9 Consider a company which has 3 plants with the production capacity as below

Plants	Product Capacity
P ₁	50
P ₂	40
P ₃	30
Total	120

Demand for the product at the company's 3 retail outlets is as shown below:

Retail Outlets	Forecasted Demand
R ₁	45
R ₂	15
R ₃	30
Total	90

The production costs at each plant is different, and the sales prices at the retail outlets vary. Taking prices, production costs, and shipping costs into consideration, the profit for producing one unit at plant i , shipping it to retail outlet j , and selling it at retail outlet j are presented in the table below :

Plants	Retail Outlets		
	R ₁	R ₂	R ₃
P ₁	10	6	7
P ₂	6	11	6
P ₃	12	7	11

Profit per unit produced at plant i and sold at retail j

- Use the least cost method to obtain the initial feasible solution.
- Solve this problem to determine the quantities that should be shipped from plant i to retail j .
- Comment on your solutions obtained above.

[5 + 7 + 3marks]

- B10 The Operations Research department at NUST has just received requests for market research studies from 3 new clients. The department is faced with the task of assigning Project leaders to each of the new research studies. Currently, 3 individuals are to carry out the research but the time required to complete each study will depend on the experience and ability of the project leaders assigned to each study. The department would like to assign these project leaders such that the total number of days required to complete the 3 projects is minimized. The time (in days) for each leader-client assignment is given below

Project leader	Client		
	1	2	3
Nyamugure	10	15	9
Sigauke	9	18	5
Madziwa	6	14	3

- Formulate the above problem as a linear programming problem. (do not solve)
- Use the Hungarian method to find the optimal assignment which will minimise project completion time and the total completion time.
- Another project leader Mr Vuma is now available for assignment to the 3 new clients with the completion times (in days) of 8, 16 and 6 days to client 1, 2 and 3 respectively. Determine the new assignment and the total completion time required by this new assignment.

[5+5+5 marks].

B11 Consider the all integer linear program

$$\text{Max } z = 8x_1 + 5x_2$$

$$\text{s.t. } \begin{aligned} x_1 + x_2 &\leq 6 \\ 9x_1 + 5x_2 &\leq 45 \end{aligned}$$

$$x_1, x_2 \geq 0 \quad x_1, x_2 \text{ integer}$$

Use the branch and bound procedure to solve this problem.

[15 marks]

END OF QUESTION PAPER