NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2107: Linear programming

December 2004

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY FOUR questions from section B

Section A: Answer all questions in this section. [40 marks]

A1 Briefly explain the following;
(a) An objective function,
(b) A non linear programming model,
(c) An optimal solution,
(d) Sensitivity Analysis,
(e) Evaluation index.

[10marks]

A2 A pilot's diet requires that the food he/she eats come from one of the four basic food groups (chocolate cake, ice cream, soda and cheese cake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola and pine apple cheese. Each brownie costs $50, each scoop of chocolate ice cream costs $20, each bottle of cola costs $30, and each pine of pineapple cheese costs $80. Each day the pilot must ingest at least 500 calories, 6g of chocolate, 10g of sugar, and 8g of fat. The nutritional content per unit of each food is shown below.

	Calories	Chocolate	Sugar	Fat
Brownies	400	3	2	2
Chocolate ice cream	200	2	2	4
Cola	150	0	4	1
Pineapple cheese	500	0	4	5

Formulate a linear programming model that can be used to satisfy the pilot's daily nutritional requirements at minimum costs.

[6 marks]

A3 Use the **Two Phase Method** to solve the following problem,

Min $z = 4x_1 + x_2$

subject to,

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

[5marks]

A4 Solve the following transportation problem whose objective is to maxmize total profit.

	Destination				
	A	B	C	D	Supply
Source 1	2	3	5	6	5
2	2	1	3	5	10
3	3	8	4	6	15
Demand	12	8	4	6	

[6marks]

A5 In a job shop operation, four jobs can be performed on any of the four machines. The number of hours required for each job on each machine is summarised in the table below. Find the assignment which will minimise the total time to complete the four jobs.

	J1	J2	J3	J4	
M1	32	18	32	26	
M2	22	24	12	16	
M3	24	30	26	24	
M4	26	30	28	20	

[5marks]

A6 Consider the all integer linear program given below

Max $5x_1 + 8x_2$

subject to,

$6x_1 + 5x_2 \leq 30$

$9x_1 + 4x_2 \leq 36$

$1x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ and integer.

(a) Graph the constraints for this problem using heavy dots to indicate all the feasible integer solutions.

(b) Find the optimal solution to the L.P Relaxation problem.

(c) Find the optimal integer solution to the above problem. **[3+3+2marks]**

Section B: Answer FOUR Questions in this section. [60 marks]

B7 Shell refinery blends four petroleum components into tree grades of gasoline; regular, premium and low lead. Management wishes to determine the optimum mix of the four components that will max profit. The maximum quantities available of each component and the cost per barrel are as follows.

Component	**Maximum Barrels Available/day**	**Cost per barrel ($)**
1	5 000	9
2	2 400	7
3	4 000	12
4	1 500	6

In order to ensure the proper blend for each gasoline grade, maximum or minimum percentages of the components in each blend have been determined. The blends as well as the selling price of each grade are given in the table below.

Grade	**Component Specifications**	**Selling price/ barrel ($)**
Regular	Not less than 40% of 1 Not more than 20% of 2 Not less than 30% of 3	$12
Premium	Not less than 40% of 3	$18
Low Lead	Not more than 50% of 2 Not less than 10% of 1	$10

Formulate a linear programming model that will maximize profit.

[15 marks]

B8 (a) Define each of the following ;
(i) Degenerate solution,
(ii) Alternate optimal solution,
(iii) Unbounded solution,
(iv) Infeasible solution. **[4 marks]**

(b) Each of the following linear programming models falls under one of the situations described in (a) above. By using the graphical method show which linear programming models labeled 1 to 4 falls in each situation described above.

Linear programming Model 1

Max $z = 3x_1 + 9x_2$

subject to,

$x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0.$

Linear programming Model 2

Max $z = 2x_1 + 4x_2$

subject to,

$x_1 + x_2 \leq 4$

$x_1 + 2x_2 \leq 5$

$x_1, x_2 \geq 0.$

Linear programming Model 3

Max $z = 2x_1 + x_2$

subject to,

$x_1 - x_2 \geq 10$

$2x_2 \leq 40$

$x_1, x_2 \geq 0.$

Linear programming Model 4

Max $3x_1 + 2x_2$

subject to,

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0.$

[11 marks]

B9 ABC has a one year contract to supply motors for all refrigerators produced by Imperial. Imperial manufactures the refrigerators at four locations as below (in thousands);

Locations	Product Capacity
L_1	50
L_2	70
L_3	60
L_4	80

ABC has three plants that are capable of producing the motors. The plants and the production capacity (in thousands) ;

Production Plants	Production capacities
P_1	100
P_2	100
P_3	150

The profit that ABC earns on each lot of 1000 units depends on which plant produced the lot and which destination it was shipped to. The following table gives the estimates of the profit ($) per unit (shipments will be made in lots of 1000 units)

Shipped to

Produced at	L_1	L_2	L_3		
P_1	7	11	8	13	
P_2	20	17	12	10	
P_3	8	18	13	16	

(a) Develop the network representation of this problem.
(b) Use the **Vogel approximation** method to obtain the initial feasible solution.
(c) Solve this problem to determine the quantities that should be shipped from plant **i** to retail **j**.
(d) Comment on your solutions obtained above.

[2+4 + 7+ 2marks]

B10 A shop needs to assign four jobs it has just received to four workers. The varying skills of the workers give rise to varying costs for performing the jobs. The table below summarises the cost data of the assignments. The data indicates that worker 1 cannot work on job 3, and worker 3 cannot work on job 4.

		Job			
		1	2	3	4
Worker	1	$50	$50	-	$20
	2	$70	$40	$20	$30
	3	$90	$30	$50	-
	4	$70	$20	$60	$70

(a) Determine the optimal assignment.
(b) Suppose that an additional (fifth) worker becomes available for performing the four jobs at the respective costs of $60, $45, $30 and $80. Which worker if any should be replaced by the new worker?
(c) The shop has just received a new a fifth job and that the respective cost of performing the job by the four current workers is $20, $10, $20 and $80 respectively. Should the new job take priority over any of the four jobs the shop has already?

[5+5+5marks]

B11 (a) Define Integer linear programming.
(b) Consider the all integer linear program,

Max $z = 5x_1 + 4x_2$
subject to,
$x_1 + x_2 \leq 5$
$10x_1 + 6x_2 \leq 45$

$x_1, x_2 \geq 0$ and integer.

Use the branch and bound procedure to solve this problem.

[2+13 marks]

END OF QUESTION PAPER