

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

BSc PART II SUPPLEMENTARY EXAMINATIONS 2001

SMA2111 MATHEMATICS FOR SCIENCE III

July 2001
3 Hours (100 marks)

This paper has 4 pages

Answer ALL questions in Section A (28 marks) and any FOUR in SECTION B (18 marks each).

Statistical Tables and graph paper are provided, however, statistical tables should not be marked or taken out of the Examination Room.

Calculators may be used.

SECTION A: Answer ALL questions in this section.

1. Write TRUE or FALSE for each of the following:

- a) The mean, median, and mode have the same value for the normal distribution. (1 mark)
- b) The median is not affected by extreme values in a data set while the mean is. (1 mark)
- c) Statistical hypotheses are always stated with respect to sample statistics and not population parameters. (1 mark)
- d) As the level of significance increases, say from 1% to 5%, the critical region shrinks/decreases. (1 mark)
- e) The probability of committing a Type I Error is equal to the level of significance in hypothesis testing. (1 mark)

2. The incomplete table below displays the distribution of the HIV status of a population in some region X.

GENDER	STATUS		Total
	HIV ⁺	HIV ⁻	
Female			180
Male	90	75	
Total	90		300

- a) Complete the table. (3 marks)
- b) Carry out an appropriate test at the 5% level of significance to test whether there is any association between HIV status and gender. (6 marks)

3. In the kinetic theory of gases, the distance X that a molecule travels before colliding with another molecule has a continuous distribution with probability density function given by

$$f(x) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{for } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where λ is some positive constant.

- a) Find the mean distance travelled by a molecule between collisions (also called its mean free path). (5 marks)
- b) Find the probability that a molecule's intercollision distance is less than half its mean free path. (4 marks)
4. A paint manufacturer wants to determine the mean drying time of a new interior wall paint. If for 12 test areas of equal size he obtained a mean drying time of 66.3 minutes and a standard deviation of 8.4 minutes, find a 95% confidence interval for the true mean, μ , assuming that drying time is normally distributed. (5 marks)

SECTION B: Answer any FOUR questions in this Section

5. a) Five specimens of a ferrous-type substance are to be used to determine if there is a significant difference between a laboratory chemical analysis and an X-ray fluorescence analysis of the iron content. Each specimen was split into two sub-specimens and the two types of analysis were applied. The following are the data showing the iron content analysis:

Analysis	Specimen				
	1	2	3	4	5
X-ray	2.0	2.0	2.3	2.1	2.4
Chemical	2.2	1.9	2.5	2.3	2.4

Assuming that the two are taken from two normal populations, carry out an appropriate t-test to test whether the methods of analysis give, on average, the same iron content. Use the 5% level of significance. (9 marks)

- b) You are now told that ten specimens were available for use and that five specimens were chosen at random and a chemical analysis was applied. An X-ray fluorescence analysis was applied on the other five specimens and the results were as shown in (a) above. Make appropriate assumptions and then carry out an appropriate t-test at the 5% level of significance to test whether the two methods of analysis give, on average, the same iron content. (9 marks)
6. Two protein diets were allocated randomly to male chicks of the same age and breed. The weight (in gms) of chicks after four weeks are as follows:

Diet	Weight (in gms)					
	150	136	144	168	152	140
High protein						
Low protein	121	109	115	106	99	

- a) Carry out a two-sample (pooled) t-test at the 5% level of significance to test whether there is a significant difference between the two protein diets. (9 marks)

- b) Carry out an Analysis of Variance (ANOVA) to test whether there is a significant difference between the two protein diets at the 5% level of significance. (9 marks)

7. Determining small quantities of calcium in the presence of magnesium is a difficult problem for the analytical chemist. Direct precipitation is not feasible. One of the procedures proposed involves the use of alcohol as a solvent. The data that follow present the results of applying the alcohol method to ten mixtures containing known quantities of CaO , x . The second row gives, in each instance, the amount of CaO recovered, y .

CaO present (x)	4.0	8.0	12.5	16.0	20.0	25.0	31.0	36.0	40.0	40.0
CaO recovered (y)	3.7	7.8	12.1	15.6	19.8	24.5	31.1	35.5	39.4	39.5

$$\sum_{i=1}^{10} x_i = 232.5; \quad \sum_{i=1}^{10} y_i = 229.0;$$

$$\sum_{i=1}^{10} x_i^2 = 6974.2; \quad \sum_{i=1}^{10} y_i^2 = 6796.7;$$

$$\sum_{i=1}^{10} x_i y_i = 6884.65$$

- a) Calculate the correlation coefficient of x and y and interpret it. (4 marks)
- b) Fit a straight line to the data by estimating the slope, β_1 and the intercept β_0 . Predict CaO recovered when CaO present is 40. (7 marks)
- c) Test $H_0: \beta_1 = 1$ versus $H_a: \beta_1 \neq 1$ at the 0.05 level of significance. (7 marks)
8. The number of births per day at a particular hospital was recorded over a period of 100 days. Use the sample data below, where x_i denotes the number of births per day and f_i denotes the number of days with x_i births, to determine if the number of births per day follows a Poisson distribution. Use $\alpha = 0.05$. Show all the steps.

i	1	2	3	4	5	6	7
x_i	0	1	2	3	4	5	6
f_i	24	20	18	11	7	5	15

(18 marks)

9. For the following data, test at the 0.01 level of significance whether or not a person's ability in chemistry and his/her interest in statistics are independent.

INTEREST IN STATISTICS	ABILITY IN CHEMISTRY		
	Low	Average	High
Low	63	42	15
Average	58	61	31
High	14	47	29

(18 marks)

END OF QUESTION PAPER.