

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA2111 - MATHEMATICS FOR SCIENCE III

This paper has 4 pages

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Time : 3 Hours

Answer **ALL** questions in Section A and **any FOUR** in Section B.**SECTION A:** Answer **ALL** questions in this Section. This section carries 28 marks.

1. The data shown below are chemical process yield readings on successive days.

127	125	131	124	129	121	142	151	160	125	125	123	120	119	128
133	137	124	142	123	121	136	140	137	125					

- (a) Use the sample mean, median and mode to guess the shape of the distribution of the yields of the chemical process.
[3+2 marks]
- (b) Use the empirical rule to check whether or not the distribution of the yields of the chemical process is normal.
[2+3 marks]
2. If the probabilities are, respectively, 0.23, 0.24, and 0.38 that a car stopped at a road block will have faulty brakes, badly worn tyres, either faulty brakes or badly worn tyres or both, what is the probability that such a car will have both faulty brakes and badly worn tyres?
[3 marks]
3. A system consists of n components and can function adequately if at least one component is in working condition. If each component is, independently, in working condition with probability 0.6, what is the probability the system functions adequately?
[3 marks]
4. The number of defects on a bolt of cloth produced by a certain process has a Poisson distribution with mean 0.4. If a random sample of 5 bolts of cloth is inspected, what is the probability that the total number of defects on the five bolts will be exactly 6?
[3 marks]
5. The life (in hours) of a certain kind of TV tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} \frac{1}{100}e^{-x/100} & \text{for } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) What is the expected (average or mean) life of the TV tube? [3 marks]
- (b) What is the probability that such a tube in a TV set will last at most 150 hours? [3 marks]
6. Experts consider high cholesterol levels to be associated with increased incidence of coronary heart disease. Suppose that the logarithm of cholesterol levels (Y) for males in a given age bracket are normally distributed with mean $\mu = 2.35$ and standard deviation $\sigma = 1.2$. If 95% of the males in this age bracket have serum cholesterol levels less than k mg/ml, what is k ? [3 marks]

SECTION B: Answer **FOUR** questions in this Section. Each question carries 18 marks.

7. A proportion 0.1 of the electronic components produced by a certain company are defective. An acceptance testing procedure is used by the purchaser to protect against lots that contain too many defective components. The procedure consists of selecting n components at random from a large batch and testing them. If at most two of the components are defective, the purchaser buys from the company.
- (a) If $n = 4$, what is the probability that the purchaser will buy from the company? [2 marks]
- (b) If $n = 40$, what is the approximate probability that the purchaser will not buy from the company? **Justify your approximation.** [5+3 marks]
- (c) If the purchaser wants to buy from the company with probability 0.05, how large should be the sample size n ? **Justify your method for solving the problem.** [5+3 marks]
8. The tensile strength of a fiber used in manufacturing cloth is of interest to the purchaser. Previous experience indicates that the mean tensile strength (μ) is 127 psi and that the standard deviation (σ) is 2 psi. A random sample of 49 fiber specimens is selected and the sample mean (\bar{X}) tensile strength determined.
- (a) Describe the sampling distribution of \bar{X} . **Justify your answer.** [4+2 marks]
- (b) What is the approximate value of the 95th percentile of the distribution of the sample mean (\bar{X}) - the average tensile strength of a random sample of 49 fiber specimens? [6 marks]

(c) Out of 10 000 random samples of 49 fiber specimens each, how many would you expect to have sample means in the interval from 126.4444 to 127.5556?

[6 marks]

9. A petroleum company may soon have to switch a large proportion of its production from a formulation containing *tetra-ethyl* lead to a lead free formulation. The best formulation is one that has a high octane number. An experiment is performed in which 10 observations on road octane number are obtained for each formulation. The data follow.

Tetra-ethyl lead	Lead free
$\bar{X}_1 = 90.70$	$\bar{X}_2 = 90.80$
$S_1^2 = 1.34$	$S_2^2 = 1.07$

(a) Do the formulations differ in their road octane numbers? Use $\alpha = 0.05$.

[8 marks]

(b) Explain how you would test your hypotheses in part (a) (above) at the 0.05 level of significance but using a confidence interval.

[4 marks]

(c) Under what assumption(s) are your conclusions in part (a) valid?

[6 marks]

10. Is cocaine deadlier than heroin? A study reported that rats with unlimited access to cocaine had poorer health, had more behavior disturbances, and died at a higher rate than did the corresponding group of rats given unlimited access to heroin. The death rates after 30 days on the study were as follows:

	% Dead at 30 days
Cocaine Group	90
Heroin Group	36

Suppose that 100 rats were used in each group. Let π_1 denotes the true death rate in the cocaine population and π_2 denotes the true death rate in the heroin population.

(a) Construct a 95% confidence interval for $\pi_1 - \pi_2$ and interpret it in terms of the problem.

[6 marks]

(b) Test the hypotheses $H_0 : \pi_1 - \pi_2 = 0$ versus $H_a : \pi_1 - \pi_2 > 0$ at the 0.05 level of significance. **Show all the steps.**

[8 marks]

(c) Under what assumptions is your confidence interval in (a) and the results of your test in (b) valid?

[4 marks]

11. A chemist is studying the relationship between X = concentration of a metal in solution (in mg/ml) and Y = absorbance. The chemist's data are summarised below.

$$\begin{array}{l} \text{Absorbance (Y)} \\ \sum_{i=1}^{10} Y_i = 6.8, \quad \sum_{i=1}^{10} Y_i^2 = 5.48 \end{array} \quad \begin{array}{l} \text{Concentration (X)} \\ \sum_{i=1}^{10} X_i = 0.32, \quad \sum_{i=1}^{10} X_i^2 = 0.013 \end{array}$$
$$\sum_{i=1}^{10} X_i Y_i = 0.265$$

- (a) Find the least squares prediction equation for the model

$$Y = \beta_0 + \beta_1 X + \epsilon.$$

- (b) Given that the estimate of the error variance is $MSE = 0.0053$, test the hypotheses: [6 marks]

$$H_0 : \beta_1 = 15 \text{ versus } H_a : \beta_1 > 15$$

at the 0.05 level of significance. Show all the steps.

- (c) Predict the true mean absorbance when the metal concentration is zero. [8 marks]

[4 marks]