NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2206

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

NUMERICAL ANALYSIS

MAY 2003

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Perform two iterations of the secant method to approximate the root of

$$f(x) = 3x + \sin x - e^x = 0$$

lying in the interval $[0, 1]$. [6]

A2. Solve the following system by the Gauss-Seidel iteration method using $x_0 = 1, y_0 = -1$ and $z_0 = 3$ as the initialization. Perform two iterations.

$$\begin{aligned} 2x + 4y - z &= -5 \\ x + y - 3z &= -9 \\ 4x + y + 2z &= 9 \end{aligned}$$

[5]

A3. The equation $x \tan x = 4$ has an infinite number of roots. To find the root near $x = 1$, the following iteration scheme may be used:

$$x_{n+1} = \arctan\left(\frac{4}{x_n}\right).$$

(a) Show that the process is linearly convergent.

(b) Starting with $x_0 = 1$, find x_3 and assess its accuracy.

[8]

A4. Use the fourth order Runge-Kutta method to solve the initial value problem

$$y' = \frac{x-y}{2}, \quad y(0) = 1$$

on $[0, 3]$ using step-size $h = 1$. [7]

A5. Using six points, apply Simpson's rule to approximate π from the formula

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}.$$

[7]

A6. Using Taylor series, establish the error term for the formula giving the derivative as

$$f'(0) \approx \frac{1}{2h}[f(2h) - f(0)].$$

[7]

SECTION B: Answer THREE questions in this section [60].

B7. The reciprocal of any real number $A \neq 0$ can be computed without dividing by using the formula

$$x_{k+1} = x_k(2 - Ax_k), \; k = 0, \; 1, \; 2, \cdots$$

(a) Derive this relation by applying the Newton-Raphson method to an appropriate function.

(b) Find $\frac{1}{7}$, with $x_0 = 0.2$ as initialization, by five iterations of the formula in (a).

(c) Show that the convergence is quadratic. [8,6,6]

B8. (a) Find an approximation to the area of the region bounded by the normal curve

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{(x/\sigma)^2/2}$$

and the x-axis on the interval $-\sigma, \sigma$ using the composite Trapezoidal rule with $n = 8$. [9]

(b) Determine a, b, c and d for the non-trivial quadrature rule of the form

$$af(-h) + bf(0) + cf(h) = hdf(h) + \int_{-h}^{h} f(x)dx$$

that is exact for polynomials of as high a degree as possible. [11]

B9. (a) Given the three points

x	x_0	x_1	x_2
$f(x)$	f_0	f_1	f_2

(i) Obtain an expression of Lagrange polynomial based on the three points x_0, x_1, x_2.

(ii) Integrate this polynomial over $[x_0, x_2]$ and hence derive the Simpson's rule for the function $f(x)$ using the three points. [4,8]

(b) The truncation error of a quadratic interpolation in an equidistant table is bounded by

$$\frac{h^3}{9\sqrt{3}} \max .|f'''(x)|.$$

Let a function f be defined by $f(x) = x^2 \ln x$. Determine [using $f(x)$] the maximum step-size, h, that can be used in the tabulation of $f(x)$ so that the error in a quadratic interpolation will be less than 10^{-5} on the interval $[5, 10]$. [8]

B10. (a) Determine the Lagrange form of the cubic interpolating function for the given discrete function with

$$f(0) = -0.5, \quad f(0.1) = 0, \quad f(0.3) = 0.2, \quad f(0.5) = 1$$

and use the result to estimate $f(0.25)$. [6]

(b) For the data given in the table below:

x	-2	-1	0	1	2
y	-19	3	1	-1	-3

(i) Without using a divided difference table, find the Newton interpolating polynomial of least degree;

(ii) Construct the divided difference table, and use it to check your polynomial to the above question. [7,7]

END OF QUESTION PAPER