NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2206

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

NUMERICAL ANALYSIS SUPPLEMENTARY EXAM

AUGUST 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Define a) rounding error b)truncation error and give an example of each. [6]

A2. Show that the Newton-Raphson method has first order convergence for a multiple root. [7]

A3. Use Simpson's rule to estimate the integral

$$I = \int_0^{\pi/4} \sqrt{1 - \sin x} dx$$

to 4 decimal places using 6 strips. [6]

A4. Solve the following initial value problem

$$x'(t) = 1 + x^2 t, \quad x(0) = 1$$

by applying the second order Taylor series method with $h = 0.1$ to estimate $x(0.2)$. [7]

A5. Determine the error term, $E_T(f,h)$, in the following numerical differentiation formula:

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + E_T(f,h).$$

[7]

A6. Solve the following linear system using Gaussian elimination with scaled partial pivoting:

$$\begin{aligned} 2x + 4y - z &= -5 \\ x + y - 3z &= -9 \\ 4x + y + 2z &= 9. \end{aligned}$$

[7]

SECTION B: Answer FOUR questions in this section [60].

B7. (a) Derive a Gaussian quadrature rule of the form

$$\int_0^4 f(x)dx \approx af(0) + bf(1) + cf(3)$$

that is exact for all polynomials of as high a degree as possible. What is the highest degree of precision?
Use this quadrature rule to evaluate

$$\frac{1}{2\pi}\int_0^3 e^{\frac{-x^2}{2}}dx.$$

[10]

(b) Use the following table for $f(x) = xe^x$ to estimate $f'(1.0)$ by a central difference method of numerical differentiation. Find also an estimate of $f''(1.0)$.

x	0.6	0.8	1.0	1.2	1.4
f(x)	1.0933	1.7804	2.7183	3.9841	5.6773

[10]

B8. (a) Use Lagrange interpolating formula inversely to obtain a root of the equation $f(x) = 0$ to 3 decimal places given that

$$f(-2) = -31, \quad f(-1) = -5, \quad f(1) = 1, \quad f(2) = 11.$$

[7]

(b) For the data given in the table below

x	-2	-1	0	1	2
y	-19	3	1	-1	-3

find the Newton interpolating polynomial of least degree. [7]

(c) What are the condition(s) for the Jacobi and Gauss-Seidel iterative processes to converge? Which of the two methods is better and why? [6]

B9. (a) Let A be a given $n \times n$ matrix and A_m be an $m \times m$ submatrix of A with $A_m \neq 0$. Prove that there exists a unique lower triangular matrix, L, with all diagonal entries being 1, and an upper triangular matrix, U, such that $A = LU$. [10]

(b) Use LU-decomposition to solve the system

$$\begin{aligned} 2x + 4y - z &= -5 \\ x + y - 3z &= -9 \\ 4x + y + 2z &= 9. \end{aligned}$$

[10]

B10. (a) Use the Bisection method to find the solution x^* of

$$x = 2^{-x}$$

correct to an accuracy of 10^{-2}, where $x^* \in [0, 1]$. How many iterations are needed to achieve an accuracy of 10^{-9}? [7]

(b) Apply the Secant method to approximate the positive solution(s) of

$$3x^2 = e^x$$

to within 10^{-4}. [6]

(c) Solve Kepler's equation

$$M = E - e\sin E$$

by 4 iterations of the Newton-Raphson method for the eccentric anomaly E, given the mean anomaly $M = 0.8$ and eccentricity $e = 0.2$. Use $E_0 = M$ as the initial estimate. [7]

END OF QUESTION PAPER