NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2206

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

NUMERICAL ANALYSIS: SUPPLEMENTARY EXAM

JULY 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Use the divided difference method to obtain a polynomial of least degree that fits the values given in the following table:

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
$\cos x$	1	0.992	0.969	0.931	0.878	0.811	0.732	0.641	0.540

Use the above values to obtain an approximate value of $\int_0^1 \cos x dx$ by Simpson's rule with 8 subintervals. [8]

A2. The following are the values of x and $\cos x$

x	0	1	2	-1	3
f(x)	-1	-1	-1	-7	5

[6]

A3. Construct a numerical integration rule of the form

$$\int_{-1}^{1} f(x)dx \approx \alpha f(-1/2) + \beta f(0) + \gamma f(1/2)$$

that is axact for all polynomials of degree equal to or less than 2. [10]

A4. (a) Find the solution of $e^x + x^4 + x = 2$ in the interval [0,1] by five iterations of the bisection method. [5]

(b) How many itertions are required for an accuracy of 10^{-10}? [5]

A5. If y_k is k-digit rounding approximation to $y = 0.d_1d_2d_3\cdots d_kd_{k+1}\cdots \times 10^n$. Show that

$$\left|\frac{y - y_k}{y}\right| \leq 0.5 \times 10^{-k+1}$$

for the case $d_{k+1} < 5$. [6]

SECTION B: Answer THREE questions in this section [60].

B6. (a) Let A be a given $n \times n$ matrix and A_m be an $m \times m$ submatrix of A with $|A_m| \neq 0$. Prove that there exists a unique lower triangular matrix, L, and an upper triangular matrix, U, such that $A = LU$. [12]

(b) Use LU decomposition to solve the system

$$\begin{bmatrix} 3 & 5 & 1 \\ 9 & 17 & 3 \\ 12 & 24 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 29 \\ 49 \end{bmatrix}$$

[8]

B7. (a) Given the initial value problem

$$y' = \frac{2}{x}y + x^2e^x, \ x \in [1,2], \ y(1) = 0$$

with exact solution $y(x) = x^2(e^x - e)$, use Taylor's method of order two with $h = 0.2$ to approximate the solution and compare it with actual values of y. [10]

(b) Apply three steps of the Runge-Kutta method of fourth order with $h = 0.2$ to the following initial value problem:

$$y' = (1 + x^{-1})y, \ y(1) = e.$$

Solve the problem analytically and compute the errors. [10]

B8. (a) Show that the Newton-Raphson method has first order convergence for a multiple root. [8]

(b) Use the bisection method to find the root accurate to within 10^{-2} on the interval [-1,-2] of the equation

$$x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$$

[6]

(c) Solve $4\cos x = e^x$ with accuracy 10^{-3}, by using the secant method with $x_0 = \frac{\pi}{4}$ and $x_1 = \frac{\pi}{2}$. [6]

B9. (a) Use the following values to construct a cubic Lagrange polynomial approximation to $f(1.09)$.

x	1.00	1.05	1.10	1.15
$f(x)$	0.1924	0.2414	0.2933	0.3492

The function being approximated is $f(x) = \log_{10} \tan x$. Use this knowledge to find a bound for the error in the approximation. [6]

(b) Using five points, apply Simpson's rule to approximate π from the formula

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}.$$

[6]

(c) Determine A, B, C and D for a formula of the form

$$Af(-h) + Bf(0) + Cf(h) = hDf'(h) + \int_{-h}^{h} f(t)dt$$

that is exact for polynomials of as high a degree as possible.
Hence approximate the integral

$$I(f) = \int_{-1}^{1} e^x dx.$$

[8]

END OF QUESTION PAPER