NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

JULY 2001 SUPPLEMENTARY EXAMINATION

SMA2207 OPERATIONS RESEARCH II

July 2001 3 Hours

This paper contains two sections, A and B.
Answer **ALL** questions in Section A.
Answer **THREE** questions from Section B.
All questions in Section B carry equal marks.

Statistical tables have been provided, and silent calculators may be used.

SECTION A : Answer ALL questions. (25 Marks)

A1. Two communication companies have bid for the contract to handle calls for Bulawayo Microproducts, where calls follow a Poisson distribution with $\lambda = 6$ per hour. The company figures a cost of \$500 per hour for calls that must wait. Poly Communications has an exponential service rate of 10 and will charge \$300 per hour. NUST Communications has an exponential service rate of 14 and will charge \$400 per hour. Who should win the bid?

(5 Marks)

A2. Explain and contrast the differences in the shortest route and minimal spanning tree problems.

(4 Marks)

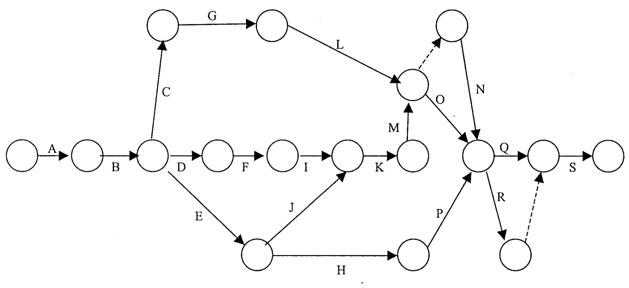
A3. Define what is meant by the terms:
i) Total float,
ii) Independent float,
iii) Free float,
in project management, and show how each would be calculated.

(6 Marks)

A4. When and why should a utility approach to decision analysis be followed?

(4 Marks)

A5. What are the immediate predacesors for each of the activities in the network shown below?



(6 Marks)

SECTION B : Answer THREE questions (all questions carry equal marks).

B1. You are starting a new job and need to buy a car so that you can get to the place of work as the public transport facilities are not adequate. The choice is between a new car costing $ 650 000 and a used car costing $ 400 000. The new car includes a guarantee for three years covering the costs of repair and the hire of a car whilst the repairs are carried out.

Five of your friends have bought used similar cars in the last few years and these cars tend to be either very reliable (VR) or absolutely awful (AW). Out of the five friend's cars two have been absolutely awful.

If a used car is very reliable then you would expect to incur repair costs over the next three years of $ 150 000 whereas, if the car turns out to be absolutely awful, the corresponding costs would be $ 350 000.

The AA offer advice on used cars as a free service to members, which you could become for a cost of $ 7 500 per year. The reliability of this advice is expressed in the following table:

	Actually VR	**Actually AW**
AA indicates OK	0.80	0.15
AA indicates bad	0.20	0.85
Totals	1.00	1.00

a) By drawing a full decision tree for the problem determine the value of the information from the AA and state your decision strategy. (22 Marks)

b) Comment on the appropriateness of the method used in (a) for this problem. (3 Marks)

B2.

a) Derive the Economic Order Quantity formula and prove that when this is used, annual order costs are equal to annual holding costs.

(6 Marks)

b) Manic Motorbikes are planning to produce 80 000 motorbikes next year, all of which will require a special type of leather seat. The seats currently cost the company $ 5 000 each with a 1% discount for orders of 10 000 or more and a 2% discount if orders are for 25 000 or more. Administrative costs associated with processing orders have been estimated as follows:

Each time an order is placed it costs $ 12 000 to cover the cost of unloading and checking the consignment. The company's cost of capital is 16% per annum and other variable stock holding costs to cover storage and insurance amount to 2% of the item's value.

i) Determine the economic order quantity and the associated total annual costs and advise management whether it is worthwhile buying in bulk in order to take advantage of the discounts offered.

(10 Marks)

ii) As an alternative to purchasing the seats from an outside supplier the company is considering setting up "in house" production. The fixed costs associated with setting up the production unit at any time are $10,000,000 and the unit variable cost of producing each seat would be $ 4 500. Once production is underway the seats could be produced at a rate of 200 000 per year.

Determine the economic production lot size and the associated total inventory cost in this case.

Write a short report to management advising the company on the optimal inventory policy.

(9 Marks)

B3. A management accountant is responsible for the time and cost control of a site-based project. Activities have been specified and normal and 'crash' completion times and costs have been estimated as follows:

Start Node	End Node	Normal total activity cost ($'000)	Normal completion time (days)	Extra cost for crash completion ($'000)	Crash completion time (days)
1	2	100	10	20	8
1	3	160	15	10	14
1	4	90	7	-	7
1	5	220	8	30	6
2	10	110	9	12	8
3	6	-	-	-	-
3	9	200	15	40	13
4	6	300	21	70	19
5	7	270	17	80	16
10	12	130	9	15	8
6	8	90	10	20	9
6	9	160	18	30	15
7	9	100	8	-	8
8	9	-	-	-	-
7	11	70	9	20	8
9	12	300	12	60	10
11	12	200	6	40	5

The project has a daily cost of $40 000 for site charges. An activity may only be undertaken at its normal or crash time (for example the activity from node 1 to 2 can take 10 days or 8 days, it **can not** take 9 days).

a) Draw the network and show the project duration and cost under normal conditions.

(8 Marks)

b) Show, with workings, the effect on the project time, cost and critical path (if any) of undertaking **all** activities at their fully crashed times.

(8 Marks)

c) Show, with workings, and explanation how the project time in (b) can be equaled and the cost reduced by crashing only some of the activities.

(9 Marks)

B4. Plane, Inc. is a manufacturer of heavy equipment and is considering introducing a new line of small steamrollers. Development is to proceed as follows.

A feasibility study will first be performed. Upon receiving a successful feasibility report, a manufacturing building is to be secured and a project leader hired. Once the building is secured, Plane will be committed to the project. Therefore an advertising group will be selected and the raw materials for the manufacturing process will be purchased. When, in addition to securing the building, and a project leader being named, a manufacturing staff will be recruited. After the manufacturing staff has been selected and the raw materials purchased, a prototype model of the steamroller will be produced. Following the completion of the prototype, work will begin on a production run of 100 steamrollers. When both the prototype model has been built and the advertising staff selected, an intensive advertising campaign will be launched. The development phase of this project will be complete with the production of the first 100 steamrollers and the initiation of the advertising campaign.

The duration of each of the activities are shown in the table below.

a) What are the immediate predecessors for each activity?

(5 Marks)

b) Draw the network for this problem.

(6 Marks)

c) How long should the project take?

(5 Marks)

d) Which activities must not be delayed if the project is to be completed in this time?

(5 Marks)

e) What will be the effect on the project duration if the manufacturing of the prototype is delayed by two weeks?

(2 Marks)

f) What is the effect on the project duration if hiring the project leader is delayed by two weeks?

(2 Marks)

Activity	Expected Duration (weeks)
Feasibility Study	6
Building Purchased	4
Project Leader Hired	3
Advertising Staff Selected	6
Materials Purchased	3
Manufacturing Staff Hired	10
Prototype Manufactured	2
Production Run of 100	6
Advertising Campaign	8

B5. The research and development department of NUST Cars is aiming to develop a prototype of a new car engine which will be designed for the anticipated market conditions of the early twenty-first century. The head of the R & D team has two main objectives (i) to minimise development costs and (ii) to design an engine which minimises carbon-dioxide (CO_2) emissions. Two competing technologies: clean burn engines and carbon reduction converters are being considered as a basis for the new design and a decision now has to be made on which technology to adopt. Each technology has been assessed under three possible scenarios : complete success, partial success and failure. The tables below show, for each scenario, the associated costs and CO_2 emissions achieved.

Clean Burn Technology

Scenario	**Development Costs**	**CO_2 Emissions per 20 000 km**
Complete Success	$200 million	3 tons
Partial Success	$150 million	8 tons
Failure	$240 million	20 tons

Carbon Reduction Converter Technology

Scenario	**Development Costs**	**CO_2 Emissions per 20 000 km**
Complete Success	$280 million	2 tons
Partial Success	$100 million	11 tons
Failure	$180 million	22 tons

It is thought that the clean burn engine technology has a 0.2 probability of being a complete success, a 0.5 probability of being a partial success, and a 0.3 probability of being a failure. For the carbon reduction converter technology these probabilities are 0.4, 0.5 and 0.1 respectively.

To help the head of the R & D team with his decision, a decision analyst has elicited utility functions for development costs and CO_2 emissions achieved. Details of these functions are shown below.

Development Costs	**Utility**	**CO_2 Emissions**	**Utility**
$280 million	0.0	22 tons	0.00
$240 million	0.4	20 tons	0.02
$200 million	0.7	11 tons	0.20
$180 million	0.8	8 tons	0.30
$150 million	0.9	3 tons	0.80
$100 million	1.0	2 tons	1.00

a) Discuss, in detail, how the decision analyst will have obtained these utilities.

(8 Marks)

b) Plot the two utility functions and explain what they show.

(6 Marks)

continued overleaf

B5 continued

c) Determine the optimal decision if the primary objective is to:

i) minimise development costs;

ii) minimise CO_2 emissions.

(8 Marks)

d) Discuss how you might decide which type of technology to use given both the objective of minimising development costs and minimising CO_2 emissions.

(3 Marks)

END OF EXAMINATION

FORMULAE SHEET

In all the following formulae λ represents the average number of arrivals per time period, μ represents the average number of services per time period, and *c* represents the number of channels.

M/M/1 Model Operating Characteristics

1. Average number of items in the queue (including zeros):

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

2. Average number of items in the queue (excluding zeros):

$$L_{q_n} = \frac{\mu}{\mu - \lambda}$$

3. Average number of items in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

4. Average time an item spends in the queue:

$$W_q = \frac{L_q}{\lambda}$$

5. Average time an item spends in the system:

$$W = \frac{L}{\lambda}$$

6. Probability of an item queuing on arrival:

$$P_w = \frac{\lambda}{\mu}$$

7. Probability of *n* units in the system:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

where P_0 is the probability the service facility is idle (i.e. $1 - \lambda/\mu$)

M/G/1 Model Operating Characteristics

1. Average number of items in the queue (including zeros):

$$L_q = \frac{2\sigma^2 + (\lambda / \mu)^2}{2(1 - \lambda / \mu)}$$

2. Average number of items in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

3. Average time an item spends in the queue:

$$W_q = \frac{L_q}{\lambda}$$

4. Average time an item spends in the system:

$$W = \frac{L}{\lambda}$$

5. Probability of an item queuing on arrival:

$$P_w = \frac{\lambda}{\mu}$$

6. Probability of the service facility being idle:

$$P_0 = 1 - \frac{\lambda}{\mu}$$

If G is the exponential distribution, then $\sigma = 1/\mu$ and the above formulas reduce to those given for the M/M/1 queue.

M/M/c Model Operating Characteristics

1. Probability of there being no items in the system:

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)}$$

2. Average number of items in the queue (including zeros):

$$L_q = \frac{(\lambda/\mu)^c \lambda\mu}{(c-1)!(c\mu-\lambda)^2} P_0$$

3. Average number of items in the system:

$$L = L_q + \frac{\lambda}{\mu}$$

4. Average time an item spends in the queue:

$$W_q = \frac{L_q}{\lambda}$$

5. Average time an item spends in the system:

$$W = \frac{L}{\lambda}$$

6. Probability of an item queuing on arrival:

$$P_w = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right) P_0$$

7. Probability of n units in the system:

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0 \qquad \text{for } n \le c$$

$$P_n = \frac{(\lambda/\mu)^n}{c!\,c^{(n-c)}} P_0 \qquad \text{for } n > c$$

M/M/1 Model with Finite Calling Population (size N) Operating Characteristics

1. Probability of there being no items in the system:

$$P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-i)!}\left(\frac{\lambda}{\mu}\right)^i}$$

2. Average number of items in the queue (including zeros):

$$L_q = N - \frac{\lambda + \mu}{\lambda}(1 - P_0)$$

3. Average number of items in the system:

$$L = N - \left(\frac{\mu + \lambda}{\lambda}\right)(1 - P_0)$$

4. Average time an item spends in the queue:

$$W_q = \frac{L_q}{(N - L)\lambda}$$

5. Average time an item spends in the system:

$$W = \frac{L}{(N - L)\lambda}$$

6. Probability of an item queuing on arrival:

$$P_w = 1 - P_0$$

7. Probability of n units in the system:

$$P_n = \frac{N!}{(N-n)!}(\lambda / \mu)^n P_0$$