

FACULTY OF APPLIED SCIENCES  
DEPARTMENT OF APPLIED MATHEMATICS  
OPERATIONS RESEARCH II

APRIL/MAY 2002

Time : 3 hours

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Candidates should attempt **ALL** questions from Section A and **ANY FOUR** questions from Section B.

SECTION A

A1. Define the following terms used in inventory control and management:

- (a) backlogging
- (b) order quantity
- (c) order point
- (d) order period

[4]

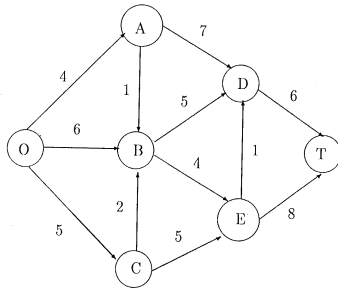
A2. Describe the minimax criterion, hence use it to solve the following decision making problem:

Category

Action	$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$
$a_1$	5	10	18	25
$a_2$	8	7	8	23
$a_3$	21	18	12	21
$a_2$	30	22	19	15

[5]

- A3. Use Dijkstra's algorithm to find the shortest route through the network, where the numbers represent actual distances between the corresponding nodes.



- A4. The purchasing cost per unit of a product is \$10 and its holding costs per unit per period is \$1.00. If the order quantity is 5 units, find the permissible range of  $p$  under optimal conditions given the following demand pdf:

$D$	0	1	2	3	4	5	6	7	8
$f(D)$	0.05	0.1	0.1	0.2	0.25	0.15	0.05	0.05	0.05

[4]

- A5. For the purpose of preparing its next year's budget, a company must gather information from its sales, production, accounting and treasury departments. The table below indicates activities and their durations. Prepare the network model of the problem and carry out the critical path calculations.

Activity	Description	Immediate Predecessor(s)	Duration (days)
A	Forecast sales volume	-	10
B	Study competitive market	-	7
C	Design item and facilities	A	5
D	Prepare production schedule	C	3
E	Estimate cost of productin	D	2
F	Set sales price	B,E	1
G	Prepare budget	E,F	14

[8]

## SECTION B

- B6. (a) Each year Automation Technology, A.T, sells 10 000 carburrators. A.T orders these carburrators from Mansen Engineering, a regional supplier of carburrators. Mansen charges \$1 500 per carburrator. Each order incurs an ordering cost of \$500. A.T believes that the demand for carburrators can be backlogged, and the cost of being short by one carburrator is \$1 500. The annual holding cost for inventory is 30 cents per dollar value of inventory.
- What is the optimal order quantity?
  - What is the maximum inventory level that will occur?
  - What is the maximum shortage that will occur?

[3,3,3]

- (b) Each year Dicks Spares sells 10 000 cases of lubrication oil. Dicks is trying to determine how many cases of lubricants should be ordered each time an order is placed with the supplier. It costs \$5.00 to process each order, and the cost of carrying a case of lubricants in inventory for one year is 20 percent of the purchase price. The lubricants supplier offers Dick Spares the schedule of quantity discounts shown in the following table, where  $q$  = number of cases ordered per order.

Number of Cases Ordered $q$	Price per Case \$
$q < 200$	4.40
$200 \leq q < 400$	4.20
$q \geq 400$	4.00

- Each time an order is placed, how many cases of lubrication oil should be ordered,
- How many orders per year will be done?
- What are the total costs of this inventory model?

[3,3,3]

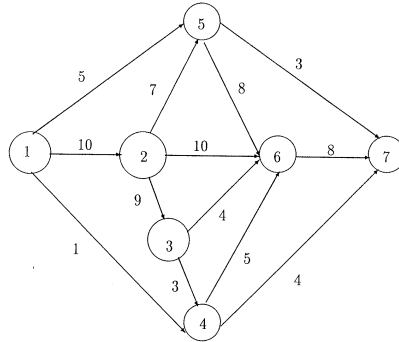
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- B7. (a) Let  $K = \$20$ ,  $D = 10000$  units,  $p = \$4$ , and  $h = \$2$ . Assuming that the demand during lead time follows a normal distribution with mean 100 and variance 4, find the optimal solution to the continuous review inventory model. [8]
- (b) Each month Max PB Garage sells 4000 litres of petrol. Each time the parent company refill's the petrol tanks, it charges the garage \$50 plus 70 cents per litre. The annual cost of holding a litre of petrol is 30 cents.
- How large should the garage's orders be?

- (ii) How many orders per year will be placed?  
 (iii) How long will it be between orders?  
 (iv) Do you think the EOQ assumptions would be satisfied? Why or why not?  
 (v) If the lead time is (i) 2 weeks, (ii) 10 weeks, what is the re-order point. Assume  
 $1 \text{ week} = \frac{1}{52} \text{ year}$ .

[2,2,2,2,2]

- B8. (a) Determine the critical path(s) for the following project.



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[9]

- (b) Given the following data for direct costs of normal and crash durations, find the first minimum cost schedule between the normal and crash points.

Activity (i,j)	Normal		Crash	
	Duration	Cost	Duration	Cost
1,2	5	100	2	200
1,4	2	50	1	80
1,5	2	150	1	180
2,3	7	200	5	250
2,5	5	20	2	40
2,6	4	20	2	40
3,4	3	60	1	80
3,6	10	30	6	60
4,6	5	10	2	20
4,7	9	70	5	90
5,6	4	100	1	130
5,7	3	140	1	160
6,7	3	200	1	240

[9]

B9. Before a new product can be introduced, the activities in the table below must be completed (all times are in weeks).

Activity	Description	Predecessors	a	b	m
A	Design the product	-	2	10	6
B	Survey of the market	-	4	6	5
C	Place orders for raw materials	A	2	4	3
D	Receive raw materials	C	1	3	2
E	Build prototype of product	A,D	1	5	3
F	Develop advertising campaign	B	3	5	4
G	Set up plant for mass production	E	2	6	4
H	Deliver product to stores	G,F	0	4	2

- (a) Draw the project diagram  
 (b) Determine the critical path  
 (c) Determine the free float and total float for each activity.  
 (d) It is now 12 weeks before the Heroes Holiday, what is the probability that the product will be in stores before the Heroes Holiday?

[3,8,4,3]

B10. (a) During the 8 - 9 pm time slot, two networks are vying for an audience of 100 million viewers. The networks must simultaneously announce the type of show they will air in that time slot. The possible choices for each network and the number of network 1 viewers (in millions) are shown in the table below.

Network 1	Network 2		
	Musical	Soap Opera	Comedy
Musical	35	15	60
Soap Opera	45	58	50
Comedy	38	14	70

Verify that this conquest has a saddle point. What is the value of the game? [9]

- (b) Two players, called Tom and Jerry, simultaneously choose the number of fingers (1 or 2) to put out. If the sum of the fingers put out by both players is odd, Tom wins \$1.00 from Jerry. If the sum of the fingers put out by both players is even, Jerry wins \$1 from Tom.
- Determine the saddle point of this game.
  - Determine the optimal strategies, and
  - value of the game.

[3,3,3]

**B11.** A fair coin is tossed and the result is shown to player 1. Player 1 must then decide whether to pass or bet. If player 1 passes, he must pay player 2 \$1. If player 1 bets, player 2 (who does not know the result of the coin toss) may either fold or call the bet. If player 2 folds, he pays player 1 \$1. If player 2 calls the bet, and the coin came up heads, player 2 pays player 1 \$2, otherwise player 1 must pay player 2 \$2.

- Formulate this as a two-person zero-sum game.
- Graphically determine the value of the game, and each player's optimal strategy.
- Show that if player 1 deviates from the optimal strategy, player 2 can ensure that player 1 earns an expected reward that is less than the value of the game.
- Show that if player 2 deviates from the optimal strategy, player 1 can ensure that he earns an expected reward that is more than the value of the game.

[2,8,4,4]

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END OF QUESTION PAPER