

**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**FACULTY OF APPLIED SCIENCES**

**DEPARTMENT OF APPLIED MATHEMATICS**

**SMA 2207: Operations Research Techniques**

**May 2005**

**Time: 3 hours**

Answer **ALL** questions from Section A and any **THREE** questions from Section B

**1** Briefly explain the importance of the following in Operations Management:

- (a) Program Evaluation and Review Technique (PERT),
- (b) Critical Path Method (CPM),
- (c) Simulation,
- (d) Shortest Route algorithm,
- (e) Queuing theory.

**[10 marks]**

**2** Considering a single item that is manufactured by a company and has a deterministic demand of  $D$  items per annum. Let  $Q$  be the quantity produced on each run,  $C_h$  is the holding cost per item, and  $C_o$  is the cost of setting a production run, and  $P$  the annual rate at which items can be produced. Derive the Economic Production Lot Size formula and show that when this is used, annual production set-up costs are equal to annual holding costs.

**[6 Marks]**

**3** Assume that the following quantity discount schedule is appropriate:

<b>Order Size</b>	<b>Discount (%)</b>	<b>Unit Cost (\$)</b>
0 to 49	0	30.00
50 to 99	5	28.50
100 or more	10	27.00

If annual demand is 120 units, ordering costs are \$20 per order, and the annual holding costs rate is 25%, what order quantity would you recommend?

**[7 marks]**

- 4 A fashion designer has just finished designing the Zimbabwe national dress. Having made the new dress he has three choices of what to do with it. Either
- 1 Making the dress himself,
  - 2 Allow someone else to make it and be paid on a royalty basis,
  - 3 Sell the rights to another designer and then paid a lump sum.
- The profit which can be expected depends on the choice he made and is shown below (in thousand of \$)

	High Sales	Medium Sales	Low Sales
Manufacture the dress	80	40	-20
Take Royalties	50	30	10
Sell all rights	20	20	20

Determine the best decision that the designer must take based on the,

- (a) Optimistic approach,
  - (b) Pessimistic approach,
  - (c) Minimax regret approach. [3+4+4 marks]
- 5 In a single channel queuing system the mean arrival rate is 0.75 units per minute and the mean service rate to process the units is 1 unit per minute. Calculate
- (a) average number of units in the waiting line ( $L_q$ )
  - (b) the average time a unit spends in the system ( $W$ ). [3+3 marks]

### SECTION B

Answer any THREE Questions in this section

- 6 Your own record company had an option of signing up the best "sungura" group known as "Cheso-Power" for a one album contract. You have divided the group's prospects into three possible outcomes;

Outcome	Probability	Expected net profit
Major success	0.1	\$100 000
Minor success	0.2	\$25 000
Dismal Failure	0.7	\$-20 000

Before making the decision whether to sign up "Cheso-Power", it would be possible for you to consult a leading expert on "sungura" music. On the basis of listening to the group's demo tapes, the expert would donate prospects as good, middling or poor. The expert's success rate in the past can be summarised as follows;

	Major Success	Minor Success	Dismal Failure
Good	0.8	0.4	0.2
Middling	0.2	0.3	0.3
Poor	0	0.3	0.5

- (a) Draw a decision tree (showing all the probabilities) and determine how much it is worth paying for the expert's advice, using the expected monetary value criterion. **[15 marks]**

- (b) Define the following as they are used in decision making analysis

- (i) States of nature,
- (ii) Minimax regret,
- (iii) Opportunity loss or regret,
- (iv) Prior probability,
- (v) Indicators.

**[5 marks]**

- 7 Time Bank has decided to modernize and refit one of its branch offices. They have awarded the contract to a local builder. The table below gives details of the activities involved together with immediately preceding activities. Also given are the normal cost of each activity, together with extra costs involved in reducing the duration of some of the activities. In addition to the cost of each activity, there are on-site cost of \$20 000 per day. The contract includes a penalty of \$10 000 per day that the project takes over 27 days.

Activity	Immediately Preceding Activity	Normal Duration (Days)	Normal Cost \$('000)	Minimum Duration (Days)	Crash Cost \$('000)
A	-	4	20	2	30
B	-	5	30	4	45
C	A	6	40	5	50
D	B	8	55	6	75
E	B	7	45	6	50
F	C	2	15	1	20
G	A	5	47	4	60
H	D	4	25	3	33
I	E,F	4	25	3	30
J	E,F	6	60	6	60
K	G,H,I	3	18	2	20
L	J,K	7	35	6	40
M	E,F	10	60	9	70
N	L	4	25	3	31

- a) Draw a network for the project and using the normal duration times find the earliest and latest event times and the critical path. Hence state the expected duration of the project. **[10 marks]**

If the normal durations are expected durations calculated from the project manager's estimates of the most likely, optimistic and pessimistic durations and if the variance of the expected duration of the project is 3.3, what is the probability that the project will be completed in 27 days or less? **[2 marks]**

- b) Specify which activities should be speeded up (by employing extra resources) and by how many days in order to:

- i) complete the project in the shortest possible time,
- ii) complete the project at minimum cost.

State any changes in the critical path in both cases. **[8 marks]**

- 8 A fast-food franchise is considering operating a drive-up window food-service operation. Assume that customer arrivals follow a Poisson probability distribution, with mean arrival rate of 24 cars per hour, and that service times follow an exponential probability distribution. Arriving customers place orders at an intercom station at the back of the parking lot and then drive up to the service window to pay for and receive their order. The following three service alternatives are being considered.

- 1 A single-channel operation where one employee fills the order and takes the money from the customer. The average service time for this alternative is 2 minutes.
- 2 A single-channel operation where one employee fills the order while a second employee takes the money from the customer. The average service time for this alternative is 1.25 minutes.
- 3 A two-channel operation with two service windows and two employees. The employee stationed at each window fills the order and takes the money for customers arriving at the window. The average service time for this alternative is 2 minutes for each channel.

Compute the following operating characteristics for each alternative, and recommend an alternative design for the fast-food franchise:

- (a) What is the probability that there are no customers in the system?
- (b) What is the average number of cars waiting for service?
- (c) What is the average time a car waits for service?

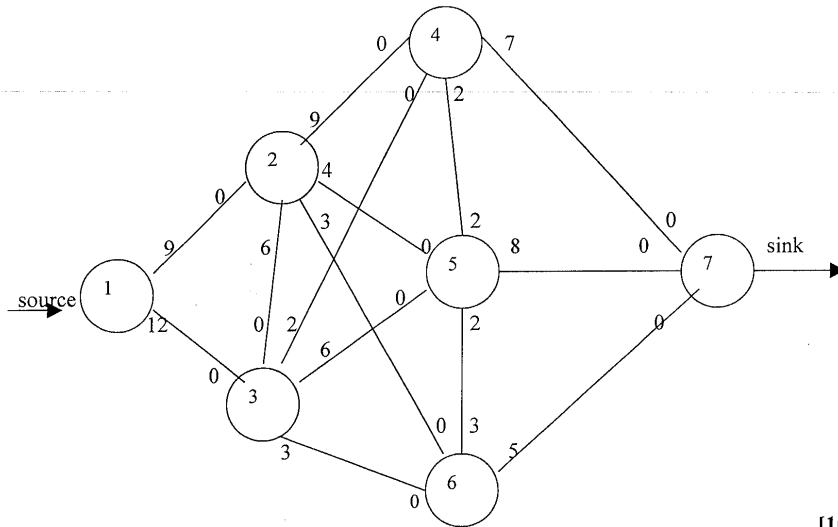
[12 marks]

The following cost information is available for the fast-food franchise

- Customer waiting time is valued at \$25 per hour to reflect that waiting time is costly to the fast-food business.
  - The cost of each employee is \$6.50 per hour.
  - To account for equipment and space, additional cost of \$20 per hour is attributed to each channel.
- (d) What is the lowest-cost design for the fast-food business?

[8 marks]

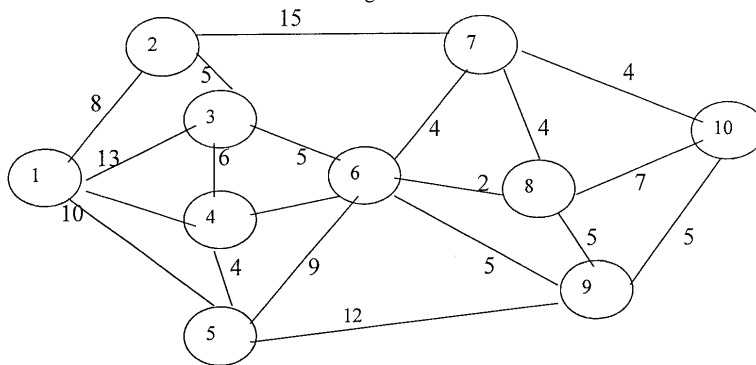
- 9 (a) The diagram below shows the electrical transmission network for Zesa. The source node (node 1) is the power generating station and the sink node (node 7) represents the destination city to which the electric power is directed. The other nodes of the network represent switching stations, at which the electrical flow of the branches of the network can be regulated. The capacities of the various branches of the network are shown on the respective branches. Find the maximum energy flow from the source to the sink.



[10 marks]

(b) Rixi taxis has identified 10 primary pickup and drop off locations for cab riders in Bulawayo. In an effort to minimise travel time and improve customer service and the utilization of company's fleet of cabs, management would like the cab drivers to take the shortest route between locations whenever possible. Using the network of roads and streets shown below ( The travel times in minutes are shown on the arcs of the network):

- (i) What is the route a driver beginning at location 1 should take to reach location 10?
- (ii) Another taxi driver has just dropped a customer at location 8, which route must he take to go back to location 1?



[5+5 marks]

**END OF QUESTION PAPER**