

SMA2215

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

B.Sc. PART II HONOURS EXAMINATIONS 2005

**TXT Part I
SBB Part II
EFW Part II**

SMA 2215 – INTRODUCTION TO APPLIED STATISTICS

MAY 2005

3 HOURS (100 Marks)

This paper has 5 pages

Answer **FOUR** questions: Question 1 in **SECTION A** (28 Marks) and **THREE** from **SECTION B** (24 Marks each). Where a question contains subdivisions, the mark value of each subdivision is indicated in brackets.

Candidates are expected to spend not more than one hour on Question 1. Calculators may be used. Statistical Tables and graph paper are provided, however, Statistical Tables should not be marked or taken out of the examination room. **GOOD LUCK!**

SECTION A (COMPULSORY)

1. (a) In each case identify the appropriate probability distribution and compute the required probability.
- (i) In a certain hospital, the probability of a multiple birth is 0.01. Suppose that you take a random sample of 20 deliveries. What is the probability of at least two multiple births? **(4 marks)**
 - (ii) In a certain city, infidelity is given as the legal reason in 80% of all divorce cases. Find the probability that eight of the next ten divorce cases filed in this city will claim infidelity as the reason. **(4 marks)**
 - (iii) A shipment of 80 burglar alarms contains 4 that are defective. If 3 of these burglar alarms are randomly selected and shipped to a customer, find the probability that the customer will get exactly one defective unit. **(3 marks)**
 - (iv) A certain type of upholstery fabric has, on average, 2 defects per 10 square metres. What is the probability that a 30-square metre bolt of this fabric will have 4 or more defects? **(5 marks)**
 - (v) A textile producer has established that a spinning machine stops randomly due to the thread breakages at an average rate of 5 stoppages per hour. What is the

probability that in a given hour, 3 stoppages will occur on this spinning machine? **(3 marks)**

(b) You have just purchased a new radio guaranteed to have an average life of two years with no trouble. Trouble-free lives of radios are normally distributed with a standard deviation of 0.5 years.

(i) What is the probability that you will have no trouble with the radio for at least 3.5 years? **(2 marks)**

(ii) What percentage of buyers of these radios would need to have them repaired within the first 15 months? **(4 marks)**

(c) If X and Y are random variables with a joint probability distribution such that, $\text{Var}(X) = 9$, $\text{Var}(Y) = 4$, and $\text{Cov}(XY) = 3$;

Find

(i) the variance of the random variable $Z = 3X - 4Y + 8$ **(1 mark)**

(ii) the covariance between the random variables $Z = 3X - 4Y$ and $W = 3X + 2Y$ **(2 marks)**

SECTION B (Answer any THREE QUESTIONS)

2. (a) A total of 210 emphysema patients entering a clinic over a one-year period were treated with one of two drugs (either the **standard drug, A**, or an experimental **compound, B**) for a period of one week. After this period, each patient's condition was rated as **greatly improved, improved, or no change**. The sample results are shown below:

Therapy	Patient's Condition		
	No Change	Improved	Greatly Improved
Standard A	20	35	45
Experimental B	15	45	50

Test, at the 5% level of significance, whether patient's condition is independent of therapy. **(8 marks)**

(b) Health authorities wish to compare the incubation period (in days) of a certain childhood disease in four different areas of a city. They take a random sample of six patients in each of the four areas and obtain the following incubation periods (in days)

Area 1	Area 2	Area 3	Area 4
13.0	12.5	7.9	8.6
12.5	8.4	8.5	10.7
18.0	9.7	10.2	9.3
15.2	10.6	11.8	13.4
17.8	16.1	12.0	11.1
19.0	14.0	15.1	12.4

Determine, at the 5% level of significance, whether there are any significant differences in the average incubation periods in the four different areas of the city. (16 marks)

3. (a) An experiment was conducted to compare the mean number of tapeworms in the stomachs of sheep that had been treated for worms against the mean number in those that were untreated. A sample of 14 worm-infected lambs was randomly divided into two groups. Seven were injected with the drug and the other seven were left untreated. After a six-month period, the lambs were slaughtered and the following worm counts were recorded:

Drug-treated sheep	18	43	28	50	16	32	13
Untreated sheep	40	54	26	63	21	37	39

Make the relevant assumptions and carry out the appropriate test at the 5% level of significance to test the hypothesis that there is no significant difference in the mean number of worms between treated and untreated lambs. Assume that the drug cannot increase the number of worms and hence use the alternative hypothesis that the mean for treated lambs is less than the mean for untreated lambs. (12 marks)

- (b) Two analysts, supposedly of identical abilities, each measure the parts per million of a certain type of chemical impurity in drinking water. It is claimed that **Analyst 1** tends to give higher readings than **Analyst 2**. To test this claim, each of six water samples is divided and analysed by both analysts separately. The following results (in ppm) are recorded:

Water Sample	1	2	3	4	5	6
Analyst 1	31.4	37.0	44.0	28.8	59.9	37.6
Analyst 2	28.1	37.1	40.6	27.3	58.4	38.9

State the underlying assumptions in this experimental design and hence test whether there is any evidence to indicate that **Analyst 1** reads higher on the average than **Analyst 2**. Use the 5% level of significance. (12 marks)

4. The laboratory of a hospital participating in the clinical trial of an antibiotic drug had to be validated to see that the laboratory personnel could accurately assay blood samples “spiked” with fixed amounts of the antibiotic. The validation consisted of the following experiment. Ten spiked samples (with amounts known only to the study investigator) were sent to the laboratory to be assayed for the amount of antibiotic present. The results of the validation experiment are shown below. (Note: The spiked samples with known amounts added, x , in micrograms per millilitre, were supplied in a blinded fashion to the laboratory.) The amounts found, y , in micrograms per millilitre, are the assay results found by the hospital laboratory.

y	0	4.5	5.0	4.8	8.9	8.9	17.0	18.2	32.6	31.5
x	0	5	5	5	10	10	20	20	40	40

- (a) Draw a scatter diagram of y against x . Fit the least squares straight line to the data and draw your fitted line on your scatter diagram. **(10 marks)**
- (b) Carry out an analysis of variance (ANOVA) to test at the 5% level of significance whether the slope is significantly different from zero. From the ANOVA table, compute the coefficient of determination, r^2 , and interpret it. **(8 marks)**
- (c) Predict the amount of antibiotic found by the hospital laboratory when the actual amount present in the blood is 15 micrograms per millilitre. Compute the standard error of the predicted value and hence find the 95% confidence interval of the predicted value. **(6 marks)**
5. An experiment was conducted to determine the effects of four different pesticides on the yield of fruit from three different varieties (V_1, V_2, V_3) of citrus. Eight trees from each variety were randomly selected from an orchard. The four pesticides were each randomly assigned to two trees of a particular variety and applications were made according to recommended levels. Yields of fruit, in bushels per tree, were obtained after the test period and are shown in the following table:

		PESTICIDE (P)			
VARIETY (V)		1	2	3	4
1		49	50	43	53
		39	55	38	48
2		55	67	53	85
		41	58	42	73
3		66	85	69	85
		68	92	62	99

- (a) Construct a two-way table of totals. **(2 marks)**

- (b) Carry out a preliminary ANOVA to test for the significance of treatment combinations at the 5% level of significance. **(8 marks)**
- (c) Carry out a complete ANOVA to test for the significance of the main effects and interaction at the 5% level of significance. **(10 marks)**
- (d) Give an appropriate table of means and appropriate standard errors for the comparisons. **(4 marks)**

6. The Celts were a vigorous race of people who once populated part of England. It is not entirely clear whether they simply died out or merged with other people who were the ancestors of those who live in England today. A study to determine whether modern Englishmen are of genetic stock comparable to the Celts was based on the comparison of maximum head breadths (in millimetres) made on unearthed Celtic skulls and a comparable number of skulls of modern-day Englishmen. The following measurements were recorded:

English:	141	148	132	138	154	142	150	146	155
	158	150	140	147	148	144	150	149	145
Celtic:	133	138	130	138	134	127	128	138	136
	131	126	120	124	132	132	125		

- (a) Construct the back-to-back **stem and leaf plots** of the data. **(5 marks)**
- (b) Construct the **box plots** of the data on the same scale. **(5 marks)**
- (c) Based on your plots in (a) and (b) above is there evidence to suggest that the modern Englishmen are of similar genetic stock to the Celts? **(4 marks)**
- (d) Carry out an appropriate statistical test to test whether there is any evidence of a significant difference in the mean breadth of skulls of the two groups. **(10 marks)**

-----*** END OF QUESTION PAPER ***-----