NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2217

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS III, SUPPLEMENTARY EXAMINATION

JULY 2001

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from section C. Tables of Laplace Transforms, some distribution functions and some formulae for Regression Analysis are given in APPENDIX 1 and APPENDIX 2 at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. A marketing research firm believes that approximately 25% of all people to whom some offer coupons were sent will respond. If a mailing of 12 coupons was conducted in a certain region,

(a) what is the probability that 10 or fewer people will respond. [4]

(b) what is the probability that more than 3 people will respond. [3]

A2. A random sample of size $n = 36$ is selected from a population with population mean equal to 40 and standard deviation equal to 12.

(a) Describe the sampling distribution of the sample mean $\bar{x}$. [2]

(b) Find $Pr(\bar{x} > 105)$. [3]

(c) Find $Pr(96 < \bar{x} < 105)$. [4]

A3. If X is normal random variable with mean μ and variance σ^2 derive the formula for calculating the sample size required to achieve a confidence interval of length L for the mean at the α significance level. [5]

A4. Suppose that a game is played with a single biased die. The die is such that even numbers are twice likely to turn up than odd numbers. In this game a player wins $20 if a 2 turns up, $40 if a 4 turns up and loses $30 if a 6 turns up. The player neither loses nor wins if any other number turns up.

(a) Find the probability distribution of the amount that a player can win. [3]

(b) Find the expected amount of money that a player can win. [2]

(c) Find the variance of the amount of money that a player can win. [3]

A5. Define the terms;

(a) population; [1]

(b) sample; [1]

(c) sample space. [2]

A6. (a) Find the Laplace transform of $-3 + 2\sin 3t - 3t^4$. [2]

(b) Find the inverse Laplace transform of $\dfrac{1}{2s^2 + 7s + 3}$. [3]

(c) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} - 4y = 6e^{-2t}, \; y(0) = -1, y'(0) = 1.$$

[7]

SECTION B: Answer TWO questions in this section [40].

B7. The voltage in a certain electrical system is a random variable with a continuous distribution for which the probability density function is given by

$$f(x) = \begin{cases} \dfrac{1}{(1+x)^2} & \text{for } x > 0, \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find the cumulative probability distribution of X and sketch it. [10]

(b) Evaluate $Pr[500 < X < 1000]$. [3]

(c) Show that the mean of this distribution does not exist. Find the median m of the distribution, such that $Pr[X \leq m] = 0.5$ [3+4]

B8. A machine tools company wants to open a hardware store. The sales profits will vary depending on the floor size of the hardware shop. The following results are available from other hardware shops that are already operating.

Floor size(X)(m^2)	35	22	27	16	28	12	40	32
Sales profit(Y)($ 1000)	20	15	17	9	16	7	22	23

(a) Compute the correlation coefficient between floor size and sales profit and comment on it. Hint $Corr(x,y) = \dfrac{cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}}$. [5]

(b) Fit the regression model $Y_i = \beta_0 + \beta_1 x_i + e_i$. What profit can be expected from a floor of size $15m^2$ [8+1]

(c) Construct an ANOVA table and test the fit of the model to the data. [6]

B9. Company officials were concerned about the length of time a particular drug retained its potency. A random sample (Sample I) of size $n_1 = 10$ bottles of the drug was chosen from the production line and analysed for potency. A second sample (Sample II) of size $n_2 = 10$ bottles was also chosen from the production line, stored for a period of one year and then analysed for potency. The following results were obtained.

	Sample I	Sample II
Mean	10.370	9.830
Variance	0.105	0.058

(a) Find a 95% confidence interval for the difference between the mean potency levels of the drugs before one year storage and those which have not been stored. [12]

(b) Test the hypothesis that the potency levels of the two categories of drugs are equal against an alternative that those which have not been stored are more potent at the $\alpha = 0.05$ level. [4]

(c) Define Type I and Type II errors. [4]

SECTION C: Answer ONE question in this section [15].

C10. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Show that the Laplace transform of $tf(t)$ is $-\dfrac{dF}{ds}$. Hence find the Laplace transform of $t\cos 2t$. [4]

(b) Show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. Hence find the Laplace transform of $H(t-3)(t-3)\cos 2(t-3)$. [4]

(c) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} + 4y = H(t-3)(t-3)\cos 2(t-3), \; y(0) = 0, y'(0) = 1.$$

[7]

C11. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = g(t), \; y(0) = 0, y'(0) = 1.$$

[6]

(b) Hence solve the initial value problem

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-2t}, \; y(0) = 0, y'(0) = 1.$$

[3]

(c) Solve the following simultaneous differential equations to find $y(t)$.

$$\begin{aligned} \frac{d^2y}{dt^2} - x &= 3t, \\ \frac{d^2x}{dt^2} - \frac{dy}{dt} &= 3, \end{aligned}$$

given that $y(0) = y'(0) = x(0) = x'(0) = 0$. [6]

APPENDIX 1: Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$u(t-c)$, where $u(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)u(t-c)$
$F(s)G(s)$	$\int_0^t f(\beta)g(t-\beta)\, d\beta$
$F(s+a)$	$e^{-at}f(t)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x = 1$ $p_x = 1-p$, for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

Simple Linear Regression:

$$\widehat{\sigma^2} = \frac{S_{yy} - \frac{(S_{xy})^2}{S_{xx}}}{n-2}, \quad Var(\beta_1) = \frac{\sigma^2}{S_{xx}}, \quad Var(\beta_0) = \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right)$$

END OF QUESTION PAPER