NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA2217: ENGINEERING MATHEMATICS III

MAY 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B and **ONE** question from sections C. A table of some distribution functions and a table of the Laplace transformations are given in the APPENDIX at the end of the question paper.

SECTION A: Answer ALL questions in this section [40].

A1. Suppose that a machine produces a defective item with probability p $(0 < p < 1)$ and produces a nondefective item with probability $q = 1 - p$. Suppose further that ten items produced by the machine are selected at random and inspected.

(a) What is the probability that all the items are nondefective?

(b) What is the probability that at least one item is defective?

[3+3]

A2. A bin contains x defective transistors (that immediately fail when put in use), y partially defective transistors (that fail after a couple of hours of use), and z acceptable transistors. A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability that it is a partially defective? [4]

A3. Suppose that the number of accidents (X) on any given weekend at a certain intersection has a Poisson distribution with mean 0.7. What is the probability that there will be at least three accidents at the intersection during the weekend? [4]

A4. The lifetime in hours (X) of a certain electronic component has the following probability density function:

$$f(x) = \begin{cases} 0.25e^{-0.25x} & x > 0, \\ 0 & x < 0. \end{cases}$$

(a) What is the mean and the median life of the electronic component?

(b) What is the probability that a new component will last at least 4 hours?

[6+4]

A5. A manufacturer produces bolts that are specified to lie between 1.19 and 1.21 cm in diameter. If his production process results in a bolt's diameter being normally distributed with mean 1.20 cm and standard deviation 0.005, what percentage of the bolts will meet the specification? [4]

A6. (a) Find the inverse Laplace transform of

(i) $\dfrac{1}{s^2+s}$

(ii) $\dfrac{1}{s^2+2s+1}$

(iii) $\dfrac{1}{s^2+2s+2}$

[2+2+2]

(b) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 6\cos(4t), \ y(0) = -1, y'(0) = 1.$$

[6]

SECTION B: Answer THREE questions in this section [45].

B7. (a) State the central limit theorem.

(b) State two assumptions of the pooled t test.

(c) A tyre company guarantees that a particular brand of a tyre has a mean (μ) useful life of 42 000 km. A consumer test agency, wishing to verify this claim, observed n=10 tyres on a test wheel that simulated normal road conditions. The lifetimes (in thousands of km) were as follows:

42, 36, 46, 43, 41, 35, 43, 45, 40, 39

Assume that the lifetimes of the tyres are normally distributed and hence test the company's claim at the 0.05 level of significance.

[4+4+7]

B8. Two different emission-control devices were tested to determine the average amount of nitric oxide being emitted by an automobile over a one-hour period of time. Twenty cars of the same model and year were selected for the study. Ten cars were equipped with Type I emission-control device, and the remaining with Type II devices. Each of 20 cars was monitored for a one-hour period to determine the amount of nitric oxide emitted. The data follow.

Type I	1.35	1.16	1.23	1.20	1.32	1.28	1.21	1.25	1.17	1.19
Type II	1.01	0.98	0.95	1.02	1.05	0.96	0.99	0.98	1.01	1.02

(a) Construct a 99% pooled t confidence interval for $\mu_1 - \mu_2$ (the difference between the mean level of emission for Type I devices and the mean level of emission for Type II devices) and interpret it in terms of the problem.

(b) Use your confidence interval in (a) above to test the hypotheses:

$$H_o: \mu_1 - \mu_2 = 0 \text{ versus } H_a: \mu_1 - \mu_2 \neq 0.$$

at the 0.01 level of significance.

[12+3]

B9. The proportion of defective sewing machines built by Pinworld Company is 0.06.

(a) In a random sample of 600 machines, find the probability that at least 30 are defective.

(b) In hopes of reducing the proportion of defective machines produced, changes were made in the manufacturing process. After the changes, a random sample of 600 machines revealed 33 defective machines. Were the changes made in the manufacturing process effective? Use $\alpha = 0.01$.

[6+9]

B10. A chemist is interested in determining the weight loss Y of a particular compound as a function of the amount of time X the compound is exposed to the air. The data below give weight losses (dependent variable) associated with $n = 12$ settings of the exposure time (independent variable).

Y	4.3	5.5	6.8	8.0	4.0	5.2	6.6	7.5	2.0	4.0	5.7	6.5
X	4	5	6	7	4	5	6	7	4	5	6	7

$$S_{YY} = 32.469;\ S_{XX} = 15.0;\ S_{XY} = 19.75;\ \sum Y = 66.1;\ \sum X = 66.$$

(a) Find the least squares prediction equation for the model

$$Y = \beta_0 + \beta_1 X + \epsilon$$

(b) Test the hypotheses:

$$H_o: \beta_1 = 0 \text{ versus } H_a: \beta_1 > 0$$

at the 0.05 level of significance.

(c) Predict the mean weight loss corresponding to zero exposure time.

[6+8+1]

B11. (a) State, clearly, the conditions under which the normal approximation to the binomial distribution are satisfactory.

(b) The quality control manager of a car parts factory would like to know whether there is a difference in the proportion of defective parts produced on different days of the work week. Random samples of 100 parts produced on each day of the week were selected with results shown below.

Result	Mon.	Tues.	Wed.	Thurs.	Frid.
#(defective)	12	7	7	10	14
#(acceptable)	88	93	93	90	86

At the 0.05 level of significance, is there a difference between the proportion of defective parts produced on Mondays and on Tuesdays?

[4+11]

SECTION C: Answer ONE question in this section [15].

Answer one question from this section.

C12. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Show that the Laplace transform of $tf(t)$ is $-\frac{dF}{ds}$. Hence find the Laplace transform of te^t. [4]

(b) Show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. Hence find the Laplace transform of $H(t-2)(t-2)e^{(t-2)}$. [4]

(c) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} - y = H(t-2)(t-2)e^{(t-2)},\ y(0) = 0, y'(0) = 1.$$

[7]

C13. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = g(t),\ y(0) = 0, y'(0) = 1.$$

[6]

(b) Hence solve the initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = e^{-2t},\ y(0) = 0, y'(0) = 1.$$

[3]

(c) Solve the following simultaneous differential equations to find $y(t)$.

$$\begin{aligned} \frac{d^2y}{dt^2} + y - x &= 1, \\ \frac{dx}{dt} - y - \frac{dy}{dt} + x &= 0, \end{aligned}$$

given that $y(0) = y'(0) = x(0) = x'(0) = 0$. [6]