NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2217

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS III

NOV/DEC 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from section C. Tables of Laplace Transforms, some distribution functions and some formulae for Regression Analysis are given in APPENDIX 1 and APPENDIX 2 at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. A graduating engineer has signed up for three job interviews. She intends to categorise the outcomes of each interview as either a success or a failure depending on whether she gets a plant trip or not.

(a) Write down the appropriate sample space of the interview outcomes. [4]

(b) What outcomes are in the event {A: Second success occurs on the third interview}? [2]

(c) What outcomes are in the event {A: First success never occurs}? [1]

A2. Use a Venn diagram to suggest a formula for $n(A \cup B \cup C)$, the number of outcomes in the union of three events. [5]

A3. Show that

$$P(x) = \frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^x, \quad x = 0, 1, 2, 3, \ldots; \quad \lambda > 0$$

qualifies as a discrete probability function. [5]

A4. In the kinetic theory of gases, the distance x, that a molecule travels before colliding with another molecule is described probabilistically by an exponential function,

$$f(x) = \left(\frac{1}{\lambda}\right) e^{-x/\lambda}, \quad x > 0; \quad \lambda > 0$$

Physicists denote the mean distance between collisions as the mean free path. What is the probability that the distance a molecule travels between consecutive collisions is less than half its mean free path. [5]

A5. (a) Describe what a confidence interval is. [1]

(b) A random sample of sample size $n = 17$ taken on a random variable X had standard deviation $s = 0.9$. Find a 95% confidence interval for the population variance σ^2 for X. [5]

A6. What sample size would be needed to produce a 95% confidence interval for the mean of length

(a) 2σ,

(b) σ, [3]

where σ is the population standard deviation. [5]

A7. (a) Find the Laplace transform of $-3 + 3e^{(-2t)} - 3t^3$. [2]

(b) Find the inverse Laplace transform of $\frac{1}{s^2 + 4s + 3}$. [3]

(c) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} - 4y = 6\cos(4t), \; y(0) = -1, y'(0) = 1.$$

[7]

SECTION B: Answer TWO questions in this section [40].

B8. A machine at a manufacturing plant fills cases with oats. The population mean weight (μ) of the cases is not known and the quality control engineer would like to obtain a 99% confidence interval for the mean weight per case. Eight cases are chosen at random and the following weights in Kilogrammes are observed.

Case	1	2	3	4	5	6	7	8
Weight(Kgs)	96.7	99.8	103.5	92.7	110.8	104.6	93.5	112.3

(a) Obtain the 99% confidence interval for μ using the data obtained. [15]

(b) Using the confidence interval in (a) above test the null hypothesis that $\mu = 100$ against the alternative hypothesis that $\mu \neq 100$. [5]

B9. A used car dealer would like to know how successful car owners were in receiving their asking price when selling their cars. The dealer selected a random sample of ten car owners who had their cars sold in a period of time. The prices are as follows:

Car owner	1	2	3	4	5	6	7	8	9	10
Selling price($)	10600	6450	5500	12890	6700	11350	8500	7540	9100	12700
Asking price($)	10950	6900	5500	12200	6700	11700	8600	8100	9200	12900

(a) Is there sufficient evidence to indicate a significant difference in the selling price and the asking price? Use a significance level of 10%. [17]

(b) What are the assumptions needed for the distributions of the two prices for you to carry out the test procedure in (a) above. [3]

B10. Let X represent the sale price($) of a certain tool sold at a hardware shop. Let Y represent the number of tools sold over a period of one month. Eight months were selected at random and the following data were recorded.

$$\Sigma x = 1022 \qquad \Sigma y = 157 \qquad \Sigma xy = 19064$$
$$\Sigma x^2 = 136938 \qquad \Sigma y^2 = 3259 \qquad n = 8$$
$$\bar{x} = 127.75 \qquad \bar{y} = 19.625$$

(a) Obtain the regression line equation for the model $\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 x$. [10]

(b) Find the correlation coefficient for X and Y and comment on it. [4]

(c) Estimate the number of tools sold if the selling price was $145.00. [1]

(d) Find a 95% confidence interval for $\widehat{\beta}_1$. [5]

Engineering Maths III: Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$H(t-c)$, where $H(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$F(s)G(s)$	$\int_0^t f(\beta)g(t-\beta)\ d\beta$
$F(s+a)$	$e^{-at}f(t)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$

APPENDIX: Some Discrete and Continuous Probability Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p,$ for $x = 1$ $p_x = 1-p,$ for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p^x)$ for $x = 0, 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $0 \leq x < 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER