NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
SMA 2217

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

ENGINNERING MATHEMATICS III

MAY 2003

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from section C. Tables of Laplace Transforms, some distribution functions and some formulae for Regression Analysis are given in APPENDIX 1 and APPEDIX 2 at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. Your next door neighbour has a rather old and temperamental burglar alarm. If someone breaks into the house, the probability of the alarm sounding is 0.95. In the last two years, it has gone off on five nights for no apparent reasons. Police records show that the chances of a home being broken into in your community on any given night are 2 in 10 000. If your neighbour's alarm goes off tomorrow night, what is the probability that his house is being broken into. [5]

A2. When taping a television commercial, the probability that a certain actor will get his lines straight on any take is 0.30. What is the probability that he will get his lines straight for first time on the sixth take. [4]

A3. A random variable X has a normal distribution with mean μ and variance σ^2. Show that the linear transformation of X given by $Z = \dfrac{X-\mu}{\sigma}$ has:

(a) mean $E(Z) = 0$ and; [3]

(b) variance $Var(Z) = 1$. [1]

A4. A fair coin is tossed 500 times. Using the normal approximation to the binomial distribution find the probability that the number of heads tossed will not differ from 250 by

(a) more than 10, [4]

(b) less 15. [5]

A5. On average 2 people from a certain population will suffer bad reaction from a certain 'immunisation day' injection. What is the probability that on a certain 'immunisation day'

(a) exactly 3 people will suffer from bad reaction from the injection. [5]

(b) more than 2 people will suffer from bad reaction from the injection. [3]

A6. (a) Derive the expression for the Laplace transform of $\frac{d^2f}{dt^2}$ in terms of the Laplace transform of f. [3]

(b) Find the inverse Laplace transform of $\frac{1}{s^2 - 4s + 3}$. [3]

(c) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} - 4y = 1 + \sin(4t), \ y(0) = -1, y'(0) = 1.$$

[6]

SECTION B: Answer TWO questions in this section [40].

B7. A company manufactures ball bearings. A sample of 34 ball bearings was taken and found to have a mean mass of 0.638 Kg with a standard deviation of 0.012 Kg. For the sample of 34 ball bearings, find

(a) a 95% confidence interval for the population mean and [10]

(b) a 95% confidence interval for the population variance. [10]

B8. (a) Define the terms Type I Error and Type II Error with the help of diagrams. [5]

(b) The mean lifetime of a sample of 100 light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the unknown population mean which is suspected to be $\mu = 1600$ hours, test the hypotheses that $\mu = 1600$ against the following alternative hypotheses:

(i) $H_1 : \mu \neq 1600$ at the $\alpha = 0.05$ significance level; [5]
(ii) $H_1 : \mu \geq 1600$ at the $\alpha = 0.05$ significance level; [5]
(iii) $H_1 : \mu \leq 1600$ at the $\alpha = 0.01$ significance level; [5]

B9. The aging of whisky in charred oak barrels brings about a number of chemical changes that enhance its taste and darken its colour. Shown in the table below is the change in the whisky quality (called the proof) over the time of storage.

Age (x) (in years)	Proof (y)
0	104.6
0.5	104.1
1	104.4
2	105.0
3	106.0
4	106.8
5	107.7
6	108.7
7	110.6
8	112.1

(a) Plot the graph of y against x and comment on the relationship between the two variables. [4]

(b) Obtain the parameters of the simple linear regression model

$$Y_i = \beta_0 + \beta_1 x_i + e_i$$

where e_i is the error term. [10]

(c) Estimate the proof of a whisky which was stored for 6.73 years. [3]

(d) Is it okay to use this model to estimate the proof of a whisky which was stored for 31 years? Comment. [3]

SECTION C: Answer ONE question in this section [15].

C10. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt.$$

(a) Show that the Laplace transform of $tf(t)$ is $-\frac{dF}{ds}$. Hence find the Laplace transform of te^{-t}. [4]

(b) Show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. Hence find the Laplace transform of $H(t-1)(t-1)e^{1-t}$. [4]

(c) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = H(t-1)(t-1)e^{1-t}, \ y(0) = 1, y'(0) = 0.$$

[7]

C11. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} + 4y = g(t), \ y(0) = 0, y'(0) = 1.$$

[6]

(b) Hence solve the initial value problem

$$\frac{d^2y}{dt^2} + 4y = \cos 2t, \ y(0) = 0, y'(0) = 1.$$

[3]

(c) Solve the following simultaneous differential equations to find $y(t)$.

$$\begin{aligned} rcl\frac{d^2y}{dt^2} - x &= 3, \\ \frac{dx}{dt} - \frac{dy}{dt} &= 3, \end{aligned}$$

given that $y(0) = y'(0) = 1$. [6]