NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2217

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA2217:ENGINEERING MATHEMATICS III SUPPLEMENTARY EXAM

July 2003

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from sections C. A table of some distribution functions is given in **APPENDIX A** and a table of Laplace Transforms is given in **APPENDIX B** at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. A coin having a probability p of coming up heads is tossed until the r^{th} head appears. What is the probability that the r^{th} head will appear on the n^{th} throw. [4]

A2. If X is a random variable having the geometric distribution with parameter p, show that the probability that X is greater than k is $(1-p)^k$. [6]

HINT: $\displaystyle\sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}$

A3. Let X be a normally distributed random variable with mean -2 and variance 0.25. Determine the value of c such that

(a) $Pr(X \geq c) = 0.2$ [4]

(b) $Pr(-2-c \leq X \leq -2+c) = 0.99$ [5]

A4. If events A and B are in a sample space S Prove that $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ [5]

A5. The probability density function of a random variable X is given by

$$f(x) = \begin{cases} cx^2 + x & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of c. [4]

(b) Find the cumulative density function of X and plot its graph. [3,2]

A6. (a) Find the Laplace transform of $-3 + 2e^{(2t)} - 3t^4$. [2]

(b) Find the inverse Laplace transform of $\frac{1}{s^2 - 4s + 3}$. [3]

(c) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 6\cos(4t), \ y(0) = 1, y'(0) = -1.$$

[7]

SECTION B: Answer TWO questions in this section [40].

B7. To determine the effectiveness of an industrial safety programme, the following data were collected over a period of a year on the average weekly loss of man hours due to accidents in 12 plants, before and after the programme was put in operation:

Plant:	1	2	3	4	5	6	7	8	9	10	11	12
Before:	50	87	37	141	59	65	24	88	25	36	50	35
After:	41	75	35	129	60	53	26	85	29	31	48	37

Use $\alpha = 0.05$ to test the hypothesis that the safety programme is not effective against the alternative hypothesis that the programme is effective. Assume that the number of accidents across the plants are normally distributed with equal variances. [20]

B8. Raw material used in the production of a synthetic fibre is stored in a place which has no humidity control. Measures of the relative humidity(%) in the storage place and the moisture content(%) of a sample of the raw material on 12 days yield the following results:

Humidity(%)(X):	46	53	37	42	34	29	60	44	41	48	33	40
Moisture Content(%)(Y):	12	14	11	13	10	8	17	12	10	15	9	13

(a) Fit a linear regression line to predict moisture content based on the relative humidity of the storage place. [15]

(b) Use the model in (a) to predict the moisture content when the relative humudity is 30%. [5]

B9. Fifteen male customers were asked which of two electric shavers, brand A and brand B they preferred. Nine of them preferred brand A.

(a) At the $\alpha = 0.05$ significance level, can you say brand A is preferred to brand B? [10]

(b) Find a 95% confidence interval for the proportion of customers who preferred brand A. [10]

SECTION C: Answer ONE question in this section [15].

C10. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt.$$

(a) Show that the Laplace transform of $tf(t)$ is $-\frac{dF}{ds}$. Hence find the Laplace transform of te^{2t}. [4]

(b) Show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. Hence find the Laplace transform of $H(t-1)(t-1)e^{2(t-1)}$. [4]

(c) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} + 9y = H(t-1)(t-1)e^{2(t-1)}, \quad y(0) = 1, y'(0) = 0.$$

[7]

C11. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt.$$

(a) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = g(t), \quad y(0) = 0, y'(0) = 1.$$

[6]

(b) Hence solve the initial value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}, \; y(0) = 0, y'(0) = 1.$$

[3]

(c) Solve the following simultaneous differential equations to find $y(t)$.

$$\begin{aligned} \frac{d^2y}{dt^2} - x &= 3, \\ \frac{dx}{dt} - \frac{dy}{dt} &= 3, \end{aligned}$$

given that $y(0) = y'(0) = x(0) = 1$.

APPENDIX A: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x = 1$ $p_x = 1-p$, for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots$	$\frac{k}{p}$	$\frac{k}{p}\left(\frac{1}{p}-1\right)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, 3, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v. *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ or $\frac{1}{\lambda}e^{-\frac{1}{\lambda}x}$ for $0 \le x < \infty$	$\frac{1}{\lambda}$ or λ	$\frac{1}{\lambda^2}$ or λ^2
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \le x \le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

APPENDIX B: Table of Laplace Transforms

$F(s)$	$f(t)$
$s^2F(s) - sf(0) - \frac{df}{dt}(0)$	$\frac{d^2f}{dt^2}$
$sF(s) - f(0)$	$\frac{df}{dt}$
$e^{-cs}/s, c > 0$	$H(t-c)$, where $H(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$
$\ln(1+1/s)$	$(1-e^{-t})/t$
$\ln[(s+k)/(s-k)]$	$2(\sinh kt)/t$
$\ln(1-k^2/s^2)$	$2(1-\cosh kt)/t$
$\ln(1+k^2/s^2)$	$2(1-\cos kt)/t$

END OF QUESTION PAPER