NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2217

DEPARTMENT OF APPLIED MATHEMATICS

SMA2217: ENGINNERING MATHEMATICS III

JUNE 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from section C. Tables of Laplace Transforms, some distribution functions and some formulae for Regression Analysis are given in APPENDIX 1 and APPEDIX 2 at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. An employment agency classifies classifies classifies clerical candidates in terms of skills and experience. The skills categories are: *Bookkeeping, switchboard and stenography,* the experience categories are: *less than 1 year, 1 to 3 years and more than 3 years.* For the 100 candidates currently in the files their skills and experience are summarised in the table below:

	Skill			
Experience	*Bookkeeping*	*Switchboard*	*Stenography*	**Total**
less than 1 year	15		30	50
1 to 3 years	5	10		20
more than 3 years			10	
Total		30		100

(a) Fill in the missing values for the table. [5]

(b) Are the two events, *bookkeeping skills* and *1 to 3 years experience*, independent? [2]

(c) Find the probability that a candidate chosen at random has skills in stenography or bookkeeping. [2]

(d) Find the probability that a candidate chosen at random has stenography skills given that the candidate has more than three years of experience. [3]

A2. (a) Find the mean and the variance of the random variable X, having a density function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

[5]

(b) If the random variable X has density function

$$f(x) = \begin{cases} 0.5x, & 0 \leq x \leq 2 \\ 0 & \text{Otherwise} \end{cases}$$

find the mean and variance of the random variable $Z = -2X + 5$ [5]

A3. A box contains four six sided dice. Three of the dice are fair (*all numbers from 1 to 6 are equally likely*) while one of the dice is biased so that 6 comes up two thirds of the time (*that is Pr(6)= 2/3, Pr(1 or 2 or 3 or 4 or 5)=1/3*). The dice can not be distinguished from one another. A die is chosen at random and tossed with the result being a 6. What is the probability that the biased die was tossed. [5]

A4. On average 2 cars enter a parking lot per minute.

(a) What is the probability that more than 4 cars will enter the parking lot in 2 minutes. [2]

(b) What is the mean and variance of the number of cars that enter the parking lot in 30 seconds. [2]

A5. A couple plans to continue having children until they have their first girl. If the probability of having a girl is 0.52 and all the births are independent what is the expected family size. [2]

A6. (a) Find the inverse Laplace transform of

(i) $\dfrac{1}{s^2 + 2s)}$

(ii) $\dfrac{1}{s^2 + 2s + 1}$

(iii) $\dfrac{1}{s^2 + 2s + 2}$ [2+2+2]

(b) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 6\cos(4t), \; y(0) = -1, y'(0) = 1.$$

[6]

A7. (a) Find the inverse Laplace transform of

(i) $\frac{1}{s^2 + 2s)}$

(ii) $\frac{1}{s^2 + 2s + 1}$

(iii) $\frac{1}{s^2 + 2s + 2}$ [2+2+2]

(b) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 17\cos(4t),\ \ y(0) = -1, y'(0) = 1.$$

[6]

SECTION B: Answer TWO questions in this section [40].

B8. An environmental chemist wants to find out the variation of pollution levels at various sites down a certain polluted river. After collecting samples from eight sites down the river the following results were obtained:

Table of Percentage of polluting chemical in water:

site	1	2	3	4	5	6	7	8
% of pollutant	24	56	42	74	44	28	34	27

Construct the 99% confidence interval for the population variance of the percentage of the polluting chemical in the water in the river. [20]

B9. A consumer testing service finds that four 5-litre cans of one brand of paint cover on the average 512 square metres with a standard deviation of 31 square metres while four cans from a different brand cover 492 square metres with a standard deviation of 26 square metres. It is desired to find out if the two brands differ in the area they cover.

(a) State the appropriate hypotheses. [5]

(b) Test these hypotheses at 5% significance level. [15]

B10. Suppose the random variable X has the normal distribution with mean μ and variance σ^2. Define $Z = \frac{X - \mu}{\sigma}$

(a) What is the transformation $Z = \frac{X - \mu}{\sigma}$ called. [2]

(b) Show that the mean of Z, $E(Z) = 0$. [2]

(c) Show that the variance of Z, $Var(Z) = 1$. [3]

(d) Show that $\bar{x}$ has mean μ and variance $\frac{\sigma^2}{n}$. [5]

(e) Show how the confidence interval for μ is constructed based on sample values $\bar{x}$, s^2, and sample size n. [10]

SECTION C: Answer ONE question in this section [15].

Answer one question from this section.

C11. Let $F(s)$ denote the Laplace Transform of $f(t)$, i.e.

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

(a) Stating clearly any assumptions you make, use the definition to find the Laplace transform of e^{-t}. [2]

(b) Show that the Laplace transform of $tf(t)$ is $-\dfrac{dF}{ds}$. Hence find the Laplace transform of te^{-t}. [3]

(c) Show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. Hence find the Laplace transform of $H(t-2)(t-2)e^{-(t-2)}$. [4]

(d) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} + y = H(t-2)(t-2)e^{-(t-2)}, \quad y(0) = 0, y'(0) = 1.$$

[6]

C12. Let $F(s)$ and $G(s)$ denote the Laplace Transforms of $f(t)$ and $g(t)$. The convolution theorem states that the Laplace Transform of $\int_0^t f(\beta)g(t=\beta)\,d\beta$ is $F(s)G(s)$.

(a) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = g(t), \quad y(0) = 0, y'(0) = 1.$$

[6]

(b) Hence solve the initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 3e^{-t}, \quad y(0) = 0, y'(0) = 1.$$

[3]

(c) Solve the following simultaneous differential equations to find $y(t)$.

$$\begin{aligned} rcl\frac{d^2y}{dt^2} + y - x &= 1, \\ \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} + x + y &= 0, \end{aligned}$$

given that $y(0) = y'(0) = x(0) = x'(0) = 0$. [6]

APPENDIX 1: Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$u(t-c)$, where $u(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)u(t-c)$
$F(s)G(s)$	$\int_0^t f(\beta)g(t-\beta)\ d\beta$
$F(s+a)$	$e^{-at}f(t)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[X^2]-(E[x])^2$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p,$ for $x=1$ $p_x = 1-p,$ for $x=0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x=0,1,\cdots,n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x=k,k+1,k+2,\cdots,n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x=1,2,\cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \dfrac{e^{-\theta}\theta^x}{x!}$ for $x=0,1,2,\cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x)=\lambda e^{-\lambda x}$ for $x>0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x)=\dfrac{1}{b-a}$ for $a\le x\le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty<x<\infty$	μ	σ^2

Simple Linear Regression:

$$\widehat{\sigma^2} = \frac{S_{yy}-\frac{(S_{xy})^2}{S_{xx}}}{n-2}, \quad Var(\beta_1)=\frac{\sigma^2}{S_{xx}}, \quad Var(\beta_0)=\sigma^2\left(\frac{1}{n}+\frac{\bar{x}^2}{s_{xx}}\right)$$

END OF QUESTION PAPER