NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 2217

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA2217:ENGINEERING MATHEMATICS III: SUPPLEMENTARY EXAMINATION

AUGUST 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY TWO** questions from Sections B and **ONE** question from sections C. A table of some distribution functions is given in **APPENDIX A** and a table of Laplace Transforms is given in **APPENDIX B** at the end of the question paper.

SECTION A: Answer ALL questions in this section [45].

A1. The ages of a sample of the people attending a training course on networking in Hanoi are:

29 20 23 22 30 32 28 23 24 27 28 31 32 33 31 28 26 25 24 23 22 26

28 31 25 28 27 34

Compute the mean and the standard deviation of the sample. [1,2]

A2. The continuous random variable X is called a exponential random variable if its density function is $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$.

Show that for this random variable:

(a) the mean $E(x) = 1/\lambda$, [3]

(b) the variance $Var(x) = 1/\lambda^2$ [5]

A3. (a) Find the area beneath a standardised normal curve between the mean $z = 0$ and the point $z = -1.26$. [3]

(b) Find the probability that a normally distributed random variable X lies more than 2 standard deviations above its mean. [3]

(c) Suppose Y is a normally distributed random variable with mean 10 and standard deviation 2.1.

(i) Find $P(Y > 11)$ [2]

(ii) Find $P(7.6 \geq Y \geq 12.2)$ [3]

A4. The mean and standard deviation of n measurements randomly sampled from a normally distributed population are 33 and 4, respectively. Construct a 95% confidence interval for the population mean (μ) when

(a) $n = 5$, [4]

(b) $n = 33$. [3]

A5. A random sample of size 150 is selected from a normal population and the number of successes is 60.

Construct a 90% confidence interval for the population proportion p. [4]

A6. (a) Find the Laplace transform of $1 + 3t - 3\cos 3t$. [2]

(b) Find the inverse Laplace transform of $\frac{1}{s^2+4s+4}$. [3]

(c) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 6e^{-3t}, \ y(0) = 1, y'(0) = -1.$$

[7]

SECTION B: Answer TWO questions in this section [40].

B7. A random sample of n observation is selected from a population with unknown mean μ and variance σ^2. For each of the following situations, specify the value of the test statistic and rejection region.

(a) $H_0: \mu = 40, \quad H_1: \mu \neq 40$; with $n = 35, \ \bar{x} = 60, \ s^2 = 64$ and $\alpha = 0.05$ [7]

(b) $H_0: \mu = 120, \quad H_1: \mu \neq 120$; with $n = 40, \ \bar{x} = 140.5, \ s = 9.6$ and $\alpha = 0.01$ [7]

(c) $H_0: \mu = 11, \quad H_1: \mu \neq 11$; with $n = 48, \ \bar{x} = 9.5, \ s = 6$ and $\alpha = 0.1$ [6]

B8. Two independent random samples are selected from populations with means μ_1 and μ_2, respectively. The sample sizes, means, and standard deviations are shown in the table.

Sample 1	Sample 2
$\bar{x} = 7.5$	$\bar{x} = 6.5$
$s = 3$	$s = 1$
$n = 45$	$n = 55$

(a) Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 0$ against the alternative hypothesis $H_1 : (\mu_1 - \mu_2) \neq 0$ at $\alpha = 0.05$ [10]

(b) Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 0.5$ against the alternative hypothesis $H_1 : (\mu_1 - \mu_2) \neq 0 + .5$ at $\alpha = 0.05$. [10]

B9. (a) Show how to construct the confidence interval for variance. [10]

(b) Show how to construct the confidence interval for the difference between two population proportions. [10]

SECTION C: Answer ONE question in this section [15].

C10. (a) Let $F(s)$ denote the Laplace Transform of $f(t)$.
Given that

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt,$$

(i) show that the Laplace transform of $e^{-kt}f(t)$ is $F(s+k)$, [3]

(ii) show that the Laplace transform of $H(t-k)f(t-k)$ is $e^{-ks}F(s)$. [3]

(b) Use Laplace transforms to solve the boundary value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = H(t-1)(t-1)e^{-2(t-1)}, \; y(0) = 1, y(1) = 0.$$

[9]

C11. Let $F(s)$ denote the Laplace Transform of $f(t)$.

(a) Given that

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt,$$

show that

$$\mathcal{L}\left(tf(t)\right) = -\frac{dF}{ds}.$$

Hence find the Laplace transform of te^{-3t}. [4]

(b) Use the convolution theorem to show that

$$\mathcal{L}^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t f(\beta)\, d\beta.$$

Hence find the inverse Laplace transform of $\frac{1}{s(s+3)^2}$. [5]

(c) Use the convolution theorem to solve the initial value problem

$$\frac{d^2y}{dt^2} + 9y = g(t), \; y(0) = 1, y'(0) = 1.$$

[6]

APPENDIX A: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p,$ for $x = 1$ $p_x = 1-p,$ for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \begin{pmatrix} x-1 \\ k-1 \end{pmatrix} p^k(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots$	$\frac{k}{p}$	$\frac{k}{p}\left(\frac{1}{p}-1\right)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, 3, \cdots$	$\frac{1}{p}$	$\frac{1}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ or $\frac{1}{\lambda}e^{-\frac{1}{\lambda}x}$ for $0 \le x < \infty$	$\frac{1}{\lambda}$ or λ	$\frac{1}{\lambda^2}$ or λ^2
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \le x \le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

APPENDIX B: Table of Laplace Transforms

$F(s)$	$f(t)$
$s^2F(s) - sf(0) - \frac{df}{dt}(0)$	$\frac{d^2f}{dt^2}$
$sF(s) - f(0)$	$\frac{df}{dt}$
$e^{-cs}/s, c > 0$	$H(t-c)$, where $H(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$
$\ln(1+1/s)$	$(1-e^{-t})/t$
$\ln[(s+k)/(s-k)]$	$2(\sinh kt)/t$
$\ln(1-k^2/s^2)$	$2(1-\cosh kt)/t$
$\ln(1+k^2/s^2)$	$2(1-\cos kt)/t$

END OF QUESTION PAPER