NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2217: Engineering Mathematics III

May 2005 **Time: 3 hours**

Answer **ALL** questions from Section A, any **TWO** questions from Section B and **ONE** question from section C. Tables of Laplace Transforms and some distribution functions are given in APPENDIX 1 and APPENDIX 2 at the end of the question paper.

SECTION A : Answer all questions in this section [45 marks]

A1 Let X be a random variable with probability density function

$$f(x)=\begin{cases}\frac{3}{4} & 0\le x\le 1\\ \frac{1}{4} & 2\le x\le 3\\ 0 & elsewhere\end{cases}$$

(a) Plot the graph of $f(x)$.

(b) Find the cumulative density function of X, $F(x)$ and plot its graph.

(c) Find $P[0.5\le X\le 2.5]$.

[3+5+2 marks]

A2 Show that $P(x)=\dfrac{1}{1+\lambda}\left(\dfrac{\lambda}{1+\lambda}\right)^x$ qualifies as a discrete probability function.

[5 Marks]

A3 Every year two teams Dembare and Bosso meet once. From results of past matches, it seems that in years when Dembare wins the probability of them winning the following year is 0.7. In years when Bosso wins the probability of them winning the following year is 0.5. There are no draws. Given that Dembare won in year 2002;

(a) Draw a tree diagram with its probabilities for the next three years up to 2005.

(b) Find the probability that Bosso will win in 2005.

(c) If Bosso wins in 2005 what is the probability that it will be their first win for at least three years.

[3+3+5marks]

A4 An unbiased coin has a single dot on one side and two dots marked on the other side. The disk and an unbiased die are thrown and the random variable X is the sum of the number of dots showing on the disk and on the die.

(a) Tabulate the probability distribution of X.

(b) Show that $P[X \geq 4 / X \leq 7] = \dfrac{8}{11}$.

(c) Write down $E[X]$ and show that $Var[X] = \dfrac{19}{6}$.

[2+3+3 marks]

A5 (a) Find the inverse Laplace transforms of;

(i) $\dfrac{1}{s^2 + 4s + 3}$ (ii) $\dfrac{1}{s^2 + 4s + 4}$ (ii) $\dfrac{1}{s^2 + 4s + 5}$.

[5 marks]

(b) Use Laplace transforms to solve the initial value problem

$$\frac{d^2 y}{dt^2} + 3\frac{dy}{dt} + 2y = 6\cos 2t - 2\sin 2t \quad \textit{given that} \quad y(0) = 0 \quad \textit{and} \quad y'(0) = 3.$$

[6 marks]

Section B

Answer any TWO questions from this section [40 marks]

B6 **(a)** Three boys Henry, Notho and Tom agree to meet at a club. Henry cannot remember whether they agreed to meet at Paparazi or Visions, and he tosses a coin to decide between the two places. Notho also tosses a coin to decide between Visions and Silver Fox. Tom tosses a coin to decide whether to go to Paparazi or not, and in this latter case he tosses again to decide between Visions and Silver fox.

Find the probability that;

(i) Henry and Notho meet.
(ii) Notho and Tom meet.
(iii) Henry, Notho and Tom all meet.
(iv) All three go to different places.
(v) At least two meet.

[12 marks]

(c) The burning time X of an experimental rocket is a random variable having a normal distribution with mean 600 seconds and standard deviation 25 seconds. Find the probability that the rocket will burn for;

(i) less that 500 seconds,

(ii) more than 640 seconds,

(iii) between 580 seconds and 630 seconds.

[8 marks]

B7 The following are the number of mistakes made in five successive days by four technicians working for a medical laboratory.

Technician 1	13	16	12	14	15
Technician 2	14	16	11	19	15
Technician 3	13	18	16	14	18
Technician 4	18	10	14	15	12

(a) Test at the 0.05 significance level whether the differences in the number of mistakes can be attributed to chance.

(b) Calculate the factor effects of each technician.

[12+8 marks]

B8 One management policy is based on the hypothesis that the more satisfied the worker is the more productive the worker will be. A scale from 1 to 10 is used to measure productivity, with 10 being assigned to the most productive worker. A worker assigns himself or herself a satisfaction score of 10 if he/she is satisfied in every aspect of the job. A sample of 10 workers were selected at random from the work force and the data collected was as follows.

Satisfaction (Y)	5	2	9	9	5	3	5	7	9	2
Productivity(X)	4	3	8	9	6	5	4	7	8	3

(a) Draw a scatter diagram of the data.

(b) What assumption on Y do you make for you to fit a linear regression model to this kind of data.

(c) Derive the least squares estimators for β_o and β_1 for the model $Y_i = \beta_o + \beta_1 x_i + e_i$

(d) Fit the model $Y_i = \beta_o + \beta_1 x_i + e_i$

[2+3+7+8marks]

Section C

Answer any ONE questions from this section [15 marks]

C11 (a) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=1+H(t-1)e^{1-t}, \quad \textit{given that} \quad y(0)=1 \textit{ and } y'(0)=-1.$$

[9 marks]

(b) Find and sketch the function f(t) which has as its Laplace transform

$$F(s)=\frac{1}{s^2}(1-3e^{-s})+\frac{2}{s^3}(e^{-s}-e^{-2s})$$

[6 marks]

C12 (a) Use the Convolution Theorem to solve the initial value problem

$$\frac{dy}{dt}+y=f(t).$$

Hence find $f(t)=e^{-t}$

[9 marks]

(d) Use the Convolution Theorem to find the inverse Laplace transform of

(i) $\frac{F(s)}{s}$ as a single integral and

(ii) $\frac{F(s)}{s^2}$ as a double integral, containing f(t), the inverse transform of F(s).

[6 marks]

END OF QUESTION PAPER

APPENDIX 1: Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$H(t-c)$, where $H(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$F(s)G(s)$	$\int_0^t f(\beta)g(t-\beta)\, d\beta$
$F(s+a)$	$e^{-at}f(t)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k}t\sin kt$

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x = 1$ $p_x = 1-p$, for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER