NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA 2217: Engineering Mathematics III

July 2005 Supplementary Exams **Time: 3 hours**

Answer **ALL** questions from Section A, any **TWO** questions from Section B and **ONE** question from section C. Tables of Laplace Transforms and some distribution functions are given in APPENDIX 1 and APPENDIX 2 at the end of the question paper.

SECTION A : Answer all questions in this section [45 marks]

A1 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} k\left(\frac{2}{x^2} - \frac{1}{2}\right) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant

(a) Show that $k = 2$ **[2 marks]**

(b) Find the cumulative distribution function F of X, and hence or otherwise find the value of t for which $F(t) = \frac{2}{3}$ **[4 marks]**

(c) Find the mean of X and show that the variance of X is approximately 0.0472 **[4 marks]**

A2 In the OK Hoza Bhora promotion there are 24 prizes to be won allocated at random to 24 winners. Phil, Nyamz and Jane are 3 of the prize winners. Of the prizes to be won 4 are cars, 8 are bicycles and 12 are watches.

(a) Show that the probability that Phil gets a car and Nyamz gets either a bicycle or a watch is $\frac{10}{69}$. **[4 marks]**

(b) Find (i) the probability that both Phil and Nyamz gets cars given that Jane gets a car.

(ii) Find the probability that either Phil or Jane (or both) gets a car **[3+4 marks]**

A3 Show that $P(x) = \frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^x$,
qualifies as a discrete probability function.

[5 Marks]

A4 An unbiased coin has a single dot on one side and two dots marked on the other side. The disk and an unbiased die are thrown and the random variable X is the sum of the number of dots showing on the disk and on the die.

(a) Tabulate the probability distribution of X.

(b) Show that $P[X \geq 4 / X \leq 7] = \frac{8}{11}$.

(c) Write down $E[X]$ and show that $Var[X] = \frac{19}{6}$.

[2+3+3 marks]

A5 (a) Find the inverse Laplace transforms of;

(i) $\frac{1}{s^2+2s}$ (ii) $\frac{1}{s^2+2s+1}$ (ii) $\frac{1}{s^2+2s+2}$.

[5 marks]

(b) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = \cos 2t - 2\sin 2t \qquad \textit{given that} \quad y(0) = 0 \quad \textit{and} \quad y'(0) = 1.$$

[6 marks]

Section B

Answer any TWO questions from this section [40 marks]

B6 (a) Three boys Henry, Notho and Tom agree to meet at a club. Henry cannot remember whether they agree to meet at Paparazi or Visions, and he tosses a coin to decide between the two places. Notho also tosses a coin to decide between Visions and Silver Fox. Tom tosses a coin to decide whether to go to Paparazi or not, and in this latter case he tosses again to decide between Visions and Silver fox.

Find the probability that;

(i) Henry and Notho meet.

(ii) Notho and Tom meet.

(iii) Henry, Notho and Tom all meet.

(iv) All three go to different places.

(v) At least two meet.

[12 marks]

(b) The burning time X of an experimental rocket is a random variable having a normal distribution with mean 600 seconds and standard deviation 25 seconds. Find the probability that the rocket will burn for;

(i) less that 500 seconds,

(ii) more than 640 seconds,

(iii) between 580 seconds and 630 seconds.

[8 marks]

B7 The following are the number of mistakes made in five successive days by four technicians working for a medical laboratory.

Technician 1	13	16	12	14	15
Technician 2	14	16	11	19	15
Technician 3	13	18	16	14	18
Technician 4	18	10	14	15	12

(a) Test at the 0.05 significance level whether the differences in the number of mistakes can be attributed to chance.

(b) Calculate the factor effects of each technician.

[12+8 marks]

B8 An experiment was conducted to compare the effects of Musimboti and Viagra on the weight loss of humans. The sex of humans was thought to be a very important factor and three replicates were used. The total weight loss of the individuals on each drug after 10 weeks are displayed in the table below.

Drug	Sex: Women	Sex: Men
Musimboti	7.6 8.8 16.4	22.2 23.4 45.6
Viagra	19.5 17.6 37.1	30.1 24.2 54.3

Analyse the data and draw conclusions. **[20 marks]**

Section C

Answer any ONE questions from this section [15 marks]

C11 (a) Use Laplace transforms to solve the initial value problem

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=2+H(t-1)e^{t-1}, \quad \textit{given that} \quad y(0)=1 \ \textit{and} \ y'(0)=-1.$$

[9 marks]

(b) Find and sketch the function f(t) which has as its Laplace transform

$$F(s)=\frac{1}{s^2}(1-3e^{-s})+\frac{2}{s^3}(e^{-s}-e^{-2s})$$

[6 marks]

C12 (a) Use the Convolution Theorem to solve the initial value problem

$$\frac{dy}{dt} + 4y = f(t).$$

Hence find y if $f(t) = \sin 2t$

[9 marks]

(c) Use the Convolution Theorem to find the inverse Laplace transform of

(i) $\frac{F(s)}{s}$ as a single integral and

(ii) $\frac{F(s)}{s^2}$ as a double integral, containing f(t), the inverse transform of F(s).

[6 marks]

END OF QUESTION PAPER

APPENDIX 1: Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$H(t-c)$, where $H(t-c) = \begin{cases} 1 & , \quad t \geq c \\ 0 & , \quad t < c. \end{cases}$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$F(s)G(s)$	$\int_0^t f(\beta)g(t-\beta)\, d\beta$
$F(s+a)$	$e^{-at}f(t)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2+k^2}$	$\sin kt$
$\frac{s}{s^2+k^2}$	$\cos kt$
$\frac{k}{s^2-k^2}$	$\sinh kt$
$\frac{s}{s^2-k^2}$	$\cosh kt$
$\frac{1}{(s^2+k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt\cos kt]$
$\frac{s}{(s^2+k^2)^2}$	$\frac{1}{2k} t \sin kt$

APPENDIX 2: Some Discrete and Continuous Probabilty Distributions

Distribution	Probability function	Mean $E(X)$	Variance $E[(x-E(x))^2]$
Discrete r.v. *probability functions*			
1.Bernoulli	$p_x = p$, for $x = 1$ $p_x = 1-p$, for $x = 0$	p	$p(1-p)$
2.Binomial	$p_x = \binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \cdots, n$	np	$np(1-p)$
3.Negative Binomial	$p_x = \binom{x-1}{k-1} p^x(1-p)^{x-k}$ for $x = k, k+1, k+2, \cdots, n$	np	$np(1-p)$
4.Geometric	$p_x = p(1-p)^{x-1}$ for $x = 1, 2, \cdots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
5.Poisson	$p_x = \frac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \cdots$	θ	θ
Continuous r.v *density functions*			
6.Exponential	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
7.Uniform	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8.Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$	μ	σ^2

END OF QUESTION PAPER