NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 3116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS IV

DECEMBER 2001

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Obtain the Euler approximation to the solution of the initial value problem

$$y' = 2y, \quad y(0) = 1$$

on $[0, 0.15]$, using $h = 0.03$. Compare with the exact solution. [5]

A2. Apply the Secant method to approximate a positive solution of

$$3x^2 = e^x$$

to within 10^{-4}. [6]

A3. Using the Taylor series, establish the error term for the formula giving the derivative as

$$f'(0) \approx \frac{f(3h) - f(-h)}{4h}.$$

[6]

A4. Prove that

$$||\mathbf{x}_k - \mathbf{x}|| \le ||B||^k ||\mathbf{x}_0 - \mathbf{x}||$$

where B is an $n \times n$ matrix with $||B|| < 1$ and

$$\mathbf{x}_k = B\mathbf{x}_{k-1} + \mathbf{c}, \quad k = 1, 2, \cdots$$

with $\mathbf{x}_0$ arbitrary, $\mathbf{c} \in \mathbb{R}^n$ and $\mathbf{x} = B\mathbf{x} + \mathbf{c}$ [5]

A5. Illustrate inverse interpolation by estimating x such that $f(x) = 0$ given that

$$f(0) = -2 \cdot 1, \quad f(1) = -0 \cdot 9, \quad f(2) = 1 \cdot 7$$

by the Lagrange form of the quadratic interpolating polynomial. [5]

A6. Apply the power series method to solve the ordinary differential equation

$$y'' - 9y = 0.$$

[6]

A7. Use the LU-decomposition method to solve the following system of linear equations:

$$\begin{aligned} 2x + y + 4z &= 13 \\ -2x + 3y + z &= -7 \\ x + y &= 2 \end{aligned}$$

[7]

SECTION B: Answer THREE questions in this section [60].

B8. (a) Derive a numerical integration formula of the form

$$\int_0^2 f(x)dx \approx af(1/2) + bf(1) + cf(3/2)$$

that is exact for all polynomials of degree ≤ 2. What is its degree of precision? [10]

(b) Find a quadrature rule of maximum degree of precision on the interval $[-1, 1]$ using two nodes. [10]

9. (a) The impressed voltage $E(t)$ in an electric circuit obeys the equation

$$E(t) = L\frac{dI}{dt} + RI(t)$$

where R is the resistance and L is the inductance. Using $L = 0.05$, $R = 2$ and the values of $I(t)$ in the table below

t	1.0	1.1	1.2	1.3	1.4
I(t)	8.2277	7.2428	5.9908	4.5260	2.9122

to compute the approximate value of $E(1 \cdot 2)$. [5]

(b) The following are values of x and $f(x)$

x	0	1	2	4
f(x)	1	3	4	5

(i) Construct a divided difference table for the above data.

(ii) Use Newton's method to find an approximating polynomial to $f(x)$. [5,5]

(c) Apply four iterations of the Gauss-Seidel iteration method, using $x_0 = y_0 = z_0 = 1$ as initialization, to solve the following system:

$$\begin{aligned} 10x - y - z &= 13 \\ x + 10y + 3z &= 36 \\ -x - y + 10z &= 35 \end{aligned}$$

[5]

B10. (a) A perfectly elastic homogeneous string of constant mass per unit length, ρ, is stretched to length L and then disturbed from its equilibrium position at time $t = 0$, and allowed to perform small transverse vibrations after release. Assuming that the tension in the string is sufficiently large to ignore the effect of gravity, derive a partial differential equation that defines this oscillatory system by considering forces acting on a small element of the string. [10]

(b) Given the steady-state temperature $u(x, y)$ in a thin square plate of side a satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with the following boundary conditions

$$u(0, y) = u(x, 0) = u(a, y) = 0, \quad u(x, a) = f(x).$$

Show that the steady-state temperature of the plate is

$$u(x, y) = \sum_{n=1}^{\infty}\left[\frac{2}{a\sinh(n\pi)}\int_0^a f(x)\sin\frac{n\pi x}{a}dx\right]\sin\frac{n\pi x}{a}\sinh\frac{n\pi y}{a}.$$

[10]

1. (a) Determine and classify all the singular points for finite values of x of the differential equation,

$$(x^2-9)^2y''+(x-3)y'+2y=0.$$

[4]

(b) Solve the Euler-Cauchy equation

$$x^2y''-xy'-3y=0$$

by assuming Frobenius series solution. [10]

(c) Apply the Euler-Cauchy method to show that an approximation to the solution of the differential equation

$$\frac{dy}{dx}=y\sin x+x$$

from x_n to $x_{n+1}=x_n+h$ can be expressed in the form

$$y_{n+1}\approx\frac{y_n(2+h\sin x_n)+2hx_n+h^2}{2-h\sin x_{n+1}}$$

Given also that $y=1$ when $x=1$, obtain values of y when $x=1{\cdot}1$ and $x=1{\cdot}2$. [6]

END OF QUESTION PAPER