NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 3116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS IV SUPPLEMENTARY

AUGUST 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Solve the following system by the Gauss-Seidel iteration method using $x_0 = 1$, $y_0 = 1$ and $z_0 = 3$ as the initialization. Perform two iterations.

$$\begin{aligned} 2x + 4y - z &= -5 \\ x + y - 3z &= -9 \\ 4x + y + 2z &= 9 \end{aligned}$$

[6]

A2. Using the Taylor series, establish the error term for the formula giving the derivative as

$$f'(0) \approx \frac{1}{2h}[f(2h) - f(0)].$$

[5]

A3. Determine the Lagrange form of the cubic interpolating function for the given discrete function with

$$f(0) = 0.5, \quad f(0.1) = 0, \quad f(0.3) = 0.2, \quad f(0.5) = 1$$

and use the result to estimate $f(0.25)$. [6]

A4. Find all the singular points of the following equation and classify them as either regular or irregular:

$$(x^2 - x - 2)y'' + xy' + y = 0.$$

[3]

A5. Given that $P_0(x) = 1$, $P_1(x) = x$ find (a) $P_2(x)$ (b)$P_3(x)$ by using the recurrence formula

$$P_{n+1}(x) = \frac{2n+1}{n+1}\, x\, P_n(x) - \frac{n}{n+1} P_{n-1}(x).$$

[5]

A6. If y is the displacement of a vibrating string of length L from its equilibrium position, y satisfies the following partial differential equation:

$$\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2},$$

where $0 < x < L$, $t > 0$ and c is a constant. If both ends of the string are fixed and the initial conditions are $y_t(x, 0) = 0$ and $y(x, 0) = f(x)$, obtain y. [10]

A7. Obtain the second order Taylor series method for the initial value problem

$$x'(t) = x + \cos t, \quad x(0) = 1.$$

Use it to estimate $x(0.2)$. [5]

SECTION B: Answer FOUR questions in this section [60].

B8. (a) Use the power series method to solve the ordinary differential equation

$$y'' - 2xy = 0.$$

[10]

(b) Use a suitable method to solve the following differential equation:

$$y'' + \frac{1}{2x} y' + \frac{1}{4x} y = 0.$$

[10]

B9. (a) Determine the quadrature formula of the form

$$\int_0^h f(x)dx \approx w_1 f(0) + w_2 f(h) + w_3 f''(0) + w_4 f''(h)$$

that is exact for polynomials of as high a degree as possible.
Hence approximate the integral

$$I(f) = \int_0^1 e^x dx.$$

[10]

(b) Use Simpson's rule to estimate the integral $\int_1^2 (\cos \pi x/2)dx$ by taking $n = 8$. [5]

(c) Consider the table for $f(x) = xe^x - \cos x$ rounded to four decimal places,

x	1.30	1.35	1.40	1.45	1.50
$f(x)$	3.7703	4.2078	4.6670	5.1818	5.7229

Using the three-point formula with (i) $h = 0.05$ and (ii) $h = 0.1$, obtain the values of $f'(1.4)$ and $f''(1.4)$. [5]

B10. (a) A perfectly elastic homogeneous string of constant mass per unit length, ρ, is stretched to length L and then disturbed from its equilibrium position at time $t = 0$, and allowed to perform small transverse vibrations after release. Assuming that the tension in the string is sufficiently large to ignore the effect of gravity, derive a partial differential equation that defines this oscillatory system by considering forces acting on a small element of the string. [10]

(b) Given the steady-state temperature $u(x, y)$ in a thin square plate of side a satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with the following boundary conditions

$$u(0, y) = u(x, 0) = u(a, y) = 0, \quad u(x, a) = f(x).$$

Show that the steady-state temperature of the plate is

$$u(x, y) = \sum_{n=1}^{\infty} \left[\frac{2}{a \sinh(n\pi)} \int_0^a f(x) \sin \frac{n\pi x}{a} dx \right] \sin \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}.$$

[10]

B11. (a) Use the Bisection method to find the solution x^* of

$$x = 2^{-x}$$

correct to an accuracy of 10^{-2}, where $x^* \in [0, 1]$.
How many iterations are needed to achieve an accuracy of 10^{-9}? [6]

(b) Apply the Secant method to approximate the positive solution(s) of

$$3x^2 = e^x$$

to within 10^{-4}. [6]

(c) Solve the following system of equations by the LU-decomposition method:

$$\begin{aligned} 3x + 5y + z &= 9 \\ 9x + 17y + 3z &= 29 \\ 12x + 24y + 13z &= 49 \end{aligned}$$

[8]

END OF QUESTION PAPER