NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 3116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS IV

DECEMBER 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Find an approximate value of $\int_1^2 x^{-1}dx$ using Simpson's rule with $h = 1/4$. Give the bound on the error. [6]

A2. Use the divided difference method to obtain a polynomial of least degree that fits the values given in the following table:

x	0	1	2	-1	3
f(x)	-1	-1	-1	-7	5

[6]

A3. Obtain the Euler approximation to the solution of the initial value problem

$$y' = 2y, \quad y(0) = 1$$

on the interval $[0, 0.15]$, using $h = 0.05$. Compare your results with the analytic solution. [6]

A4. Use Lagrange interpolating formula inversely to obtain a root of the equation $f(x) = 0$ to 3 decimal places given that

$$f(-2) = -31, \quad f(-1) = -5, \quad f(1) = 1, \quad f(2) = 11.$$

[6]

A5. Determine A, B, C and D for a formula of the form

$$Af(-h) + Bf(0) + Cf(h) = hDf'(h) + \int_{-h}^{h} f(t)dt$$

that is exact for polynomials of as high a degree as possible.
Hence approximate the integral

$$I(f) = \int_{-1}^{1} e^x dx.$$

[8]

A6. Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, 0 < x < 3, t > 0$ given that $u(0,t) = u(3,t) = 0, u(x,0) = 5\sin 4\pi x - 3\sin 8\pi x + 2\sin 10\pi x, |u(x,t)| < M.$ [8]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Apply the Secant method to approximate the positive solution(s) of

$$3x^2 = e^x$$

to within 10^{-4}. [6]

(b) Use the bisection method to find the smallest positive root of

$$\sin 2x - e^{x-1} = 0$$

with a relative accuracy of $0 \cdot 5\%$ after finding a good starting interval (x_0, x_1). [7]

(c) Use Newton-Raphson's method to approximate, to within 10^{-3}, the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1,0)$.[**Hint:**Minimize $[d(x)]^2$, where $d(x)$ represents the distance from (x, x^2) to $(1,0)$.] [7]

B8. (a) Apply Gaussian Elimination method with scaled partial pivoting to solve the system

$$\begin{aligned} 2x + y + z + 3w &= 7 \\ x + 4y + 3z + w &= 9 \\ 2x - y - 5z + w &= -3 \\ 3x - 4y + 2z + 8w &= 9 \end{aligned}$$

[6]

(b) Prove that

$$||\mathbf{x}^{(k)} - \mathbf{x}|| \leq ||B||^k ||\mathbf{x}^{(0)} - \mathbf{x}||$$

where $B \in \mathbb{R}^{n \times n}$ with $||B|| < 1$ and

$$\mathbf{x}^{(k)} = B\mathbf{x}^{(k-1)} + \mathbf{c}, \quad k = 1,\ 2, \cdots$$

with $\mathbf{x}^{(0)}$ arbitrary, $\mathbf{c} \in \mathbb{R}^n$, and $\mathbf{x} = B\mathbf{x} + \mathbf{c}$. [6]

(c) Consider the set of equations:

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{aligned}$$

Suppose the true solutions of these equations x, y and z and the iterative values after r steps of the Jacobi process are x_r, y_r and z_r where

$$x_r = x + \alpha_r \qquad y_r = y + \beta_r \qquad z_r = z + \gamma_r.$$

Let $E_r = \max\{|\alpha_r|, |\beta_r|, |\gamma_r|\}$, i.e. the largest error after r steps.

(i) Prove that

$$\begin{aligned} |\alpha_{r+1}| &\leq \left|\frac{a_{12}}{a_{11}}\right| |\beta_r| + \left|\frac{a_{13}}{a_{11}}\right| |\gamma_r| \\ |\beta_{r+1}| &\leq \left|\frac{a_{21}}{a_{22}}\right| |\alpha_r| + \left|\frac{a_{23}}{a_{22}}\right| |\gamma_r| \\ |\gamma_{r+1}| &\leq \left|\frac{a_{31}}{a_{33}}\right| |\alpha_r| + \left|\frac{a_{32}}{a_{33}}\right| |\gamma_r| \end{aligned}$$

(ii) Comment on the convergence scheme if

$$\begin{aligned} |a_{11}| &> |a_{12}| + |a_{13}| \\ |a_{22}| &> |a_{21}| + |a_{23}| \\ |a_{33}| &> |a_{31}| + |a_{32}| \end{aligned}$$

[8]

B9. (a) Solve the following initial value problem

$$y'(x) = 1 + xy^2, \quad y(0) = 1$$

(i) by the Euler-Cauchy method;
(ii) by the Fourth Order Runge-Kutta method.
using $h = 0.1$ to estimate $x(0.2)$. Compare the results. [6,7]

(b) Derive the following rule for estimating the third derivative

$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

and determine the error term. [7]

B10. (a) Write the boundary conditions for a vibrating string of length L for which

(i) the ends $x = 0$ and $x = L$ are fixed,
(ii) the initial shape is given by $f(x)$,
(iii) the initial velocity distribution is given by $g(x)$,
(iv) the displacement at any point x at time t is bounded. [4]

(b) Solve the boundary value problem

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, \quad u(0, y) = 8e^{-3y}.$$

[6]

(c) Suppose that a unit square plate has three sides kept at temperature zero, while the fourth side is kept at temperature u_1, i.e. $u(0, y) = u(1, y) = u(x, 0) = 0, u(x, 1) = u_1$. Show that the steady-state temperature everywhere in the plate is given by

$$u(x, y) = \frac{2u_1}{\pi}\sum_{m=1}^{\infty}\frac{1 - cos m\pi}{m \sinh m\pi}\sin m\pi x \sinh m\pi y.$$

[10]

END OF QUESTION PAPER