NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 3116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SUPPLEMENTARY EXAM:ENGINEERING MATHEMATICS IV

JULY 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Construct the divided difference table for the data

x	0	1	2	4
f(x)	1	3	4	5

and use it to form the interpolating polynomial of degree 3. [6]

A2. Use Simpson's rule with $n = 8$ to approximate the integral

$$\int_1^2 \cos\left(\frac{\pi x}{2}\right) dx.$$

[6]

A3. If y is the displacement of a vibrating string of length L from its equilibrium position, y satisfies the following partial differential equation:

$$\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2},$$

where $0 < L$, $t > 0$ and c is a constant. If both ends of the string are fixed and the initial conditions are $y_t(x, 0) = 0$ and $y(x, 0) = F(x)$, obtain y. [8]

A4. Obtain the Euler approximation to the solution of the initial value problem

$$y' = 2y, \quad y(0) = 1$$

on the interval $[0, 0.15]$, using $h = 0.05$. Compare your results with the analytic solution.

[6]

A5. Use Lagrange interpolating formula inversely to obtain a root of the equation $f(x) = 0$ to 3 decimal places given that

$$f(-2) = -31, \quad f(-1) = -5, \quad f(1) = 1, \quad f(2) = 11.$$

[6]

A6. Determine A, B, C and D for a formula of the form

$$Af(-h) + Bf(0) + Cf(h) = hDf'(h) + \int_{-h}^{h} f(t)dt$$

that is exact for polynomials of as high a degree as possible.
Hence approximate the integral

$$I(f) = \int_{-1}^{1} e^x dx.$$

[8]

SECTION B: Answer THREE questions in this section [60].

B7. (a) (i) Verify that when Newton-Raphson method is used to compute $\sqrt{R}$, the sequence of iterates is defined by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right).$$

(ii) Show that if the sequence $\{x_n\}$ is defined as above, then

$$x_{n+1}^2 - R = \left(\frac{x_n^2 - R}{2x_n}\right)^2.$$

[5,5]

(b) Use the bisection method to find the root of

$$\ln\left(\frac{1+x}{1+x^2}\right) = 0$$

in the interval $(0.99, 1.01)$. What is the other root?

[5]

(c) Calculate an approximate value for $4^{3/4}$ using three steps of the secant method with $x_0 = 3$ and $x_1 = 2$.

[5]

B8. (a) Solve the following initial value problem

$$y'(x) = 1 + xy^2, \quad y(0) = 1$$

(i) by the Euler-Cauchy method;

(ii) by the Fourth Order Runge-Kutta method.

using $h = 0.1$ to estimate $x(0.2)$. Compare the results. [6,7]

(b) Derive the following rule for estimating the third derivative

$$f'''(x) \approx \frac{1}{2h^3}[f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)]$$

and determine the error term. [7]

B9. (a) Write the boundary conditions for a vibrating string of length L for which

(i) the ends $x = 0$ and $x = L$ are fixed,

(ii) the initial shape is given by $f(x)$,

(iii) the initial velocity distribution is given by $g(x)$,

(iv) the displacement at any point x at time t is bounded. [4]

(b) Solve the boundary value problem

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, \quad u(0, y) = 8e^{-3y}.$$

[6]

(c) Suppose that a unit square plate has three sides kept at temperature zero, while the fourth side is kept at temperature u_1, i.e. $u(0, y) = u(1, y) = u(x, 0) = 0, u(x, 1) = u_1$. Show that the steady-state temperature everywhere in the plate is given by

$$u(x, y) = \frac{2u_1}{\pi}\sum_{m=1}^{\infty}\frac{1 - cos m\pi}{m \sinh m\pi}\sin m\pi x \sinh m\pi y.$$

[10]

B10. (a) A trough of length L has a cross-section in the shape of a semi-circle of radius r. When filled with water to within a distance h of the top, the volume V of the water is given by

$$V = L[0.5\pi r^2 - r^2 \arcsin(h/r) - h(r^2 - h^2)^{\frac{1}{2}}].$$

Suppose $L = 10$ metres, $r = 1$ foot and $V = 12.4$ cubic metres. Use the bisection method to find the depth of water in the trough to within 0.01 metres. [7]

(b) For a system $A\mathbf{x} = \mathbf{b}$, the residual is given by $\mathbf{r} = A\mathbf{e}$ where $\mathbf{e}$ is the error in the computed solution $\tilde{\mathbf{x}}$. Show that the error bounds of the relative error, $\frac{||\mathbf{e}||}{||\mathbf{x}||}$ in terms of the relative residual $\frac{||\mathbf{r}||}{||\mathbf{b}||}$ and the condition number $\kappa(A)$ are

$$\frac{1}{\kappa(A)} \cdot \frac{||\mathbf{r}||}{||\mathbf{b}||} \leq \frac{||\mathbf{e}||}{||\mathbf{x}||} \leq \kappa(A) \cdot \frac{||\mathbf{r}||}{||\mathbf{b}||}$$

given that $\kappa(A) \geq 1$. [6]

(c) Use the following values to construct a cubic Lagrange polynomial approximation to $f(1.09)$.

x	1.00	1.05	1.10	1.15
$f(x)$	0.1924	0.2414	0.2933	0.3492

The function being approximated is $f(x) = \log_{10} \tan x$. Use this knowledge to find a bound for the error in the approximation. [7]

END OF QUESTION PAPER