NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 3116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS IV

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. (a) Define an ill-conditioned system. [2]

(b) What is the condition number of a matrix. [2]

(c) Give the distinction between interpolation and extarapolation. [3]

A2. Determine the langrange form of the cubic interpolating function for the given discrete function with

$$f(0) = -0.5, \quad f(0.1) = 0, \quad f(0.3) = 0.2, \quad f(0.5) = 1$$

and use the result to estimate $f(0.25)$. [7]

A3. Solve the following linear system using Gaussian elimination with scaled partial pivoting:

$$\begin{aligned} 2x + 4y - z &= -5 \\ x + y - 3z &= -9 \\ 4x + y + 2z &= 9 \end{aligned}$$

[6]

A4. If y_k is a k-digit rounding approximation to $y = 0.d_1d_2d_3 \cdots d_kd_{k+1} \cdots \times 10^n$. Show that

$$\left|\frac{y - y_k}{y}\right| \leq 0.5 \times 10^{-k+1}$$

for the case $d_{k+1} \geq 5$. [6]

A5. Show that the Newton-Raphson method has first order convergence for a double root. [7]

A6. Solve the boundary value problem

$$\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial^2 y}{\partial t^2}$$

with

$$y(0,t) = 0, \quad y(L,t) = 0, \quad y(x,0) = f(x), \quad y_t(x,0) = 0, \quad |y(x,t)| < M.$$

[7]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Find a bound for the number of iterations of the Bisection method needed to achieve an approximation with an accuracy of 10^{-3} to the solution of $x^3+x-4=0$ lying in the interval $[1, 4]$. Solve the equation by 5 iterations of the bisection method. [7]

(b) Solve Kepler's equation

$$M = E - e \sin E$$

by 4 iterations of the Newton-Raphson method for the eccentric anomaly E, given the mean anomaly $M = 0.8$ and eccentricity $e = 0.2$. Use $E_0 = M$ as the initial estimate. [7]

(c) Calculate an approximate value for $4^{\frac{3}{4}}$ using three steps of the secant method with $x_0 = 3$ and $x_1 = 2$. [6]

B8. (a) Using five points, apply Simpson's rule to approximate π from the formula

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}.$$

[5]

(b) Construct a numerical integration rule of the form

$$\int_{-1}^{1} f(x)dx \approx \alpha f(-1/2) + \beta f(0) + \gamma f(1/2)$$

that is axact for all polynomials of degree equal to or less than 2. [7]

(c) Determine the quadrature formula of the form

$$\int_{a}^{b} f(x)dx \approx w_0 f(a) + w_1 f(b) + w_2 f'(a) + w_3 f'(b)$$

that is exact for polynomials of as high a degree as possible. [8]

B9. (a) Given the initial value problem

$$y' = \frac{2}{x}y + x^2 e^x, \; x \in [1, 2], \; y(1) = 0$$

with exact solution $y(x) = x^2(e^x - e)$, use Taylor's method of order two with $h = 0.2$ to approximate the solution and compare it with actual values of y. [10]

(b) Apply three steps of the Runge-Kutta method of fourth order with $h = 0.2$ to the following initial value problem:

$$y' = (1 + x^{-1})y, \; y(1) = e.$$

Solve the problem analytically and compute the errors. [10]

B10. (a) Solve the following differential equation by Frobenius' method:

$$y'' + \frac{1}{2x}y' + \frac{1}{4x}y = 0.$$

[10]

(b) A bar of length l whose entire surface is insulated including its ends at $x = 0$ and $x = L$ has initial temperature $f(x)$. The temperature u behaves according to the partial differential equation

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} \quad (0 < x < l, t > 0)$$

and also satisfies the following conditions:

$$u(0, t) = 20, \quad u(l, t) = 30 \qquad t > 0$$

and that

$$u(x, 0) = f(x) \qquad 0 < x < l$$

Determine the subsequent temperature of the bar. [10]

END OF QUESTION PAPER