NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

JULY 2001 SUPPLEMENTARY EXAMINATION

SMA : 4103 **INTRODUCTION TO FLUID MECHANICS**

TIME 3 HOURS

Answer **ALL** questions in **Part A** and any **THREE** questions in **Part B**.

PART A : Answer **ALL** questions. [46 marks]

1. Let

$$\phi(x,y,z) = -\frac{q}{\sqrt{x^2+y^2+z^2}}$$

be the potential function for an irrotational flow. Find the velocity components as functions of x, y and z. **[4]**

2. Let the two-dimensional fluid velocity be given by $v_1 = \frac{kx_2x_1}{1+kx_2t}$, $v_2 = 0$. Find

   a) the streamline at time t, passing through the point (α_1, α_2) **[4]**

   b) the pathline for the particle which was at (β_1, β_2) at t=0. **[4]**

   c) the streakline at time t, for the particles which passed through (α_1, α_2) at time $\tau \leq t$. **[4]**

3. Show that

$$w(z) = \sec^2 z$$

represents a flow in the strip

$$y > 0, \quad 0 < x < \frac{\pi}{2}.$$

Sketch the streamlines. **[4:2]**

4. In a two-dimensional incompressible flow the fluid velocity components are given by

$$v_x = x - 4y, \quad v_y = -y - 4x.$$

   a) Show that the flow satisfies the continuity equation and obtain the expression for the stream function. **[6]**

   b) Calculate the vorticity of the flow. What can be said about the flowfield? **[3]**

5 A source, for a two-dimensional flow, with the strength a and a vortex with the strength b are located at the origin. Determine the equation for velocity potential and the stream function. **[7]**

6 A two-dimensional uniform flow past a cylinder of radius a can be modeled by the stream function $\psi = U(r - \frac{a^2}{r})\sin\theta$. Show that the flow is irrotational. Use Bernoulli's equation to show that the pressure at any point on the cylinder surface is given by

$$p = p_0 + \frac{1}{2}\rho U^2(1 - 4\sin^2\theta),$$

where p_0 is the pressure in undisturbed uniform flow. **[8]**

PART B : Answer any **THREE** questions. Each question carries 18 marks.

7 a) Write down the expressions for $\bar{f}(\bar{z})$ and $\bar{f}\left(\frac{a^2}{z}\right)$ for a real if

(i) $f(z) = i\ln(z - z_0)$ and (ii) $f(z) = \frac{e^{i\alpha}}{z - z_0}$. **[6]**

b) Show that the two complex potentials

$$w_1 = \cosh^{-1}\left(\frac{z}{c}\right) \text{ and } w_2 = \frac{i\pi z^2}{2c^2}$$

each represent flow between the two branches of the hyperbola $x^2 - y^2 = \frac{c^2}{2}$, with $\psi = \frac{\pi}{4}$ on one branch. Say whether the potentials represent the same flow and sketch the streamlines of the flow(s). **[12]**

8 a) Show that $\psi(x, y) = Axy$ satisfies $\nabla^2\psi = 0$ and that the potential function $\phi = A(x^2 - y^2)$ gives the same velocity field. **[6]**

b) A flow field is represented by the potential function

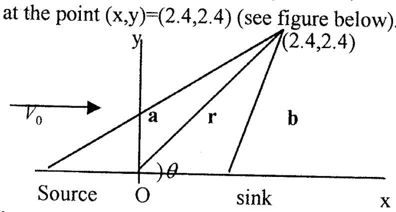
$$\phi = r^{1/2}\cos\frac{\theta}{2}.$$

(i) Find the corresponding stream function **[3]**

(ii) Verify that the flow is incompressible **[3]**

(iii) Calculate the radial and tangential components of acceleration of a particle moving in the flow field. **[6]**

9 a) Consider a source and a sink of equal strength Q=2.8 cubic meters per second placed in a uniform stream of velocity V_0 at $(-2,0)$ and $(2,0)$ respectively. What is the potential at the point (x,y)=(2.4,2.4) (see figure below).



[8]

b) A circular tank, with radius r_0, containing water is rotated, about its axis of rotation, in a large body of water at a constant angular velocity ω. Assuming that the fluid is inviscid and incompressible (assuming viscous effects are negligible near the solid wall), find the velocity distribution, vorticity and circulation in the fluid inside and outside the tank. [10]

10 a) Explain briefly what is meant by **convective** acceleration,

$$\mathbf{a} = \frac{D\mathbf{v}}{Dt} = \frac{d}{dt}\mathbf{v}(x, y, z, t).$$

Represent this in component form. [4]

b) Suppose the wall $y = 0$ is fixed, and the rigid wall $y = a$ moves at steady speed V in its plane. Solve the Navier-Stokes equations for the case ρ = constant to show that a possible flow is $v = \frac{Vy}{a}\mathbf{i}$. [4]

b) A reservoir has a narrow outlet pipe at the bottom on one side and is full of water(assumed inviscid and incompressible) to height h above the pipe. Let the cross-sectional area of the pipe be a and A(h) be the area of the free surface. Show, stating clearly any assumptions made, that the velocity of the liquid leaving the reservoir is approximately $\sqrt{2gh}$. [9]