NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

DECEMBER 2001 EXAMINATION

SMA : 4103 **INTRODUCTION TO FLUID MECHANICS**

TIME 3 HOURS

Answer **ALL** questions in **Part A** and any **THREE** questions in **Part B**.

PART A : Answer **ALL** questions. [40 marks]

1. Let $\mathbf{v}$ be a velocity field for a two-dimensional incompressible fluid that has no component in the z – direction. The velocity component v_y at any point is given by

$$v_y = y^2 - x^2$$

Find the v_x -component of velocity. **[4]**

2. Find the equation of the streamline for the steady two-dimensional flow passing through the point (1,2,3) given that

$$\mathbf{v} = (3ax, 4ay, -7az),$$

where a is a constant. **[6]**

3. Referred to rectangular coordinates (x, y, z), a fluid particle at any time t has the position given by

$$x = a, \quad y = bt^2, z = 0,$$

where a and b are constants. Change to polar cylindrical coordinates and find the velocity field

$$\mathbf{v} = v_r \hat{\mathbf{e}}_r + v_\theta \hat{\mathbf{e}}_\theta + w \hat{\mathbf{e}}_z$$

and its magnitude $|\mathbf{v}|$. **[5:2]**

4. Show that

$$w(z) = \sec^2 z$$

represents a flow in the strip

$$y > 0, \quad 0 < x < \frac{\pi}{2}.$$

Sketch the streamlines. **[3:2]**

5. The velocity potential for a line dipole of moment μ placed at a point $\mathbf{a}$ is given by

$$\phi = -\frac{\mu(\mathbf{r}-\mathbf{a})}{2\pi|\mathbf{r}-\mathbf{a}|^2}.$$

Verify that this potential satisfies Laplace's equation. **[6]**

6 Given the flow

$$[illegible] \quad v_r = -\frac{5}{r}, \quad v_\theta = 0.$$

Show that this flow satisfies the continuity equation for incompressible flow and determine the vorticity. **[6]**

7 Air flows steadily through the horizontal nozzle. The areas at the nozzle inlet and the nozzle outlet are $1000 cm^2$ and $200 cm^2$ respectively. Calculate the pressure difference between the inlet and the outlet if the inlet speed is $10 m/\sec$. Give your answer in kPa. **[6]**

PART B : Answer any **THREE** questions. Each question carries 20 marks.

8. a) Determine the scalar potential function of the vector function
$\mathbf{a} = 2xy\mathbf{i} + x^2\mathbf{j} + 3z\mathbf{k}$ **[6]**

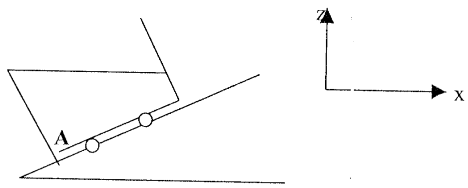
b) A flow field is represented by the potential function

$$\phi = r^{1/2}\cos\frac{\theta}{2}.$$

(i) Find the corresponding stream function **[3]**
(ii) Verify that the flow is incompressible **[3]**
(iii) Calculate the radial and tangential components of acceleration of a particle moving in the flow field. **[6]**

9. a) (i) An open tank filled with water is accelerated vertically upward at a constant $5 m/\sec^2$. Find the pressure at a point whose depth from the surface of the water is $5 m$. **[5]**

(ii) This open tank moves with a constant acceleration of $2 m/\sec^2$ in the direction making an angle of 30^o with the positive x-axis. Calculate the pressure in the corner of the tank, that is, at point **A** before and after acceleration. What can



be said about the two answers? **[5]**

b) (i) Show that for the velocity field

$$u = u(y,z), \quad v = w = 0$$

the Navier-Stokes equations, in the absence of body forces, reduce to

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{\mu}\frac{dp}{dx}=constant$$

where μ is the viscosity of the fluid and p is pressure. **[5]**

(ii) For a pipe of cross section given by $\frac{y^2}{a^2}+\frac{z^2}{b^2}=1$, find A and B if

$$u=A\left[\frac{y^2}{a^2}+\frac{z^2}{b^2}\right]+B.$$ **[5]**

10. a) Determine the vorticity in the following flow fields:
    (i) u=kx, v=-ky, w=0
    (ii) a forced vortex
    (iii) a free vortex
    (iv) potential flow around a circular cylinder with its axis normal to the stream direction. **[4:3:3:4]**

    b) Use the Bernoulli equation to find the pressure distribution for the flow of a linearly viscous fluid given above in 10a)(i). **[6]**

11. An incompressible velocity field is given by

$$u=a(x^2-y^2),\quad v=-2axy,\quad w=0$$

where a is a constant.

a) Determine under what conditions is this velocity field a solution to the Navier-Stokes momentum equation. Hence find the resulting pressure distribution when z-axis is pointing vertically upward. **[10]**

b) Show that a stream function exists for this flow. Find it and plot it. **[6]**

c) Show that a velocity potential exists for this flow and find it. **[4]**