NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

DECEMBER 2002 EXAMINATION

SMA : 4103 INTRODUCTION TO FLUID MECHANICS

TIME 3 HOURS

Answer **ALL** questions in **Part A** and any **THREE** questions in **Part B**.

PART A : Answer **ALL** questions. [40 marks]

1. The velocity field for a two-dimensional incompressible flow is given in polar coordinates by

$$v_r = r\cos\frac{\theta}{2},\ v_\theta = r\sin\frac{\theta}{2},\ 0 < \theta < 2\pi.$$

Find the corresponding stream function for this flow. Hence sketch the streamlines. **[4: 3]**

2. The differential form of the equation of continuity can be used to evaluate the relative rate of change of density of a fluid particle as it moves through a flow. Show that

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla.\mathbf{v}\ .$$

What is the physical significance of $\nabla.\mathbf{v}$. **[4:2]**

3. a) Determine whether the flow given by $\mathbf{v} = (4x^2 + 3y)\mathbf{i} + (3x - y)\mathbf{j} + 30\mathbf{k}$ is
   (i) incompressible **[3]**
   (ii) irrotational **[3]**

   b) Find the expression for vorticity for the following flows in the $r\theta$-plane:
   (i) a forced vortex **[4]**
   (ii) a free vortex. **[4]**

4. Consider a frictionless, incompressible flow described by velocity field

$$\mathbf{v} = Ax\mathbf{i} - Ay\mathbf{j}.$$

Let the only body force involved in the flow be the gravitational force and $p = p_0$ at $(x, y, z) = (0,0,0)$. Determine an expression for the pressure field, $p(x, y, z)$. **[7]**

5. Determine the scalar potential function of the vector field
   $\mathbf{a} = 2xy\mathbf{i} + x^2\mathbf{j} + 3z^2\mathbf{k}$. **[6]**

PART B : Answer any **THREE** questions. Each question carries 20 marks.

6. a) A velocity field in cylindrical coordinates is given by

$$\mathbf{v} = \left(\frac{q}{2\pi r} + U\cos\theta\right)\mathbf{e}_r - U\sin\theta\,\mathbf{e}_\theta\,,$$

where $q = 200m^2/s$ and $U = 10m/s$. Show that this is a possible incompressible flow. Locate the stagnation points in the flow. **[5:3]**

b) Consider the velocity field for steady inviscid flow from left to right over a circular cylinder, of radius a, given by

$$\mathbf{v} = U\cos\theta\left[1-\left(\frac{a}{r}\right)^2\right]\mathbf{e}_r - U\sin\theta\left[1+\left(\frac{a}{r}\right)^2\right]\mathbf{e}_\theta\,.$$

Determine the expressions for the acceleration of a fluid particle moving along the stagnation streamline, $\theta = \pi$, and that along the cylinder surface, $r = a$. Find the points where these accelerations reach maximum and minimum values. **[12]**

7. a) In attempting to solve the Navier-Stokes equations, it is necessary to impose appropriate physical conditions on the boundaries of the fluid domain. The boundary conditions imposed on a solid boundary and free surface are some of them. Describe these briefly giving appropriate examples. **[4]**

b) Consider a steady, frictionless and incompressible flow over a stationary circular cylinder of radius a given by

$$\mathbf{v} = U\left\{\left[\left(\frac{a}{r}\right)^2 - 1\right]\cos\theta\,\mathbf{e}_r + \left[\left(\frac{a}{r}\right)^2 + 1\right]\sin\theta\,\mathbf{e}_\theta\right\}.$$

(i) What is the physical significance of the difference between this formula and the one in 6b). **[3]**

(ii) For the flow along the streamline forming the cylinder surface, $r = a$, express the components of the pressure gradient in terms of the angle θ.

Plot speed, v, as a function of r along the radial line $\theta = \dfrac{\pi}{2}$ for $r > a$. **[8:5]**

8. a) (i) A steady laminar flow exists in a conduit, with all the streamlines parallel to the walls. Using the momentum equations, determine the simplified differential equations which describe the flow. **[8]**

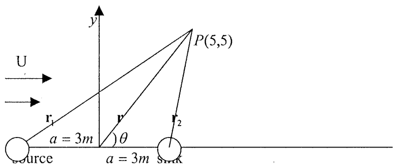
(ii) If the conduit is horizontal, and the pressure variation normal to the walls is neglected write down the resulting equation governing the flow. **[4]**

b) The flow field in a forced vortex is given by $\mathbf{v} = \omega r\mathbf{e}_\theta$, where $\omega = 10,/s$ and r is in meters. Assume a frictionless fluid with $\rho = 1000kg/m^3$. Express the radial pressure gradient, $\dfrac{\partial p}{\partial r}$, as a function of r. Hence evaluate the pressure change between $r_1 = 1m$ and $r_2 = 2m$. **[8]**

9. a) A two-dimensional incompressible flow is described by $\mathbf{v} = v_\theta \mathbf{e}_\theta = \omega r \mathbf{e}_\theta$; $v_r = 0$, in which ω is a constant. Sketch this flow and show that it satisfies the continuity equation. Also, evaluate the vorticity of the flow. **[7]**

b) Let $\phi = \ln(x^2 + y^2)^{\frac{1}{2}}$ be the velocity potential of a two-dimensional irrotational incompressible flow defined everywhere except at the origin. Find the stream function for this flow. **[6]**

c) A diagram of the source and sink of equal strength of $Q = 0.5m^3/s$ is shown below. What is the value of the stream function at $(x, y) = (5,5)$ for the combined flows, that is, the source, sink and the uniform flow parallel to the line joining the source and the sink.



[7]

10. a) Consider an incompressible flow described by the velocity field

$$\mathbf{v} = (u, v, w) = \left[a(x^2 - y^2), v(\text{unknown}), b\right],$$

where a and b are constants. Determine the form of the velocity component v. **[5]**

b) Determine under what conditions the flow described by

$$\mathbf{v} = (a(x^2 - y^2), -2axy, 0)$$

is a solution to the Navier-Stokes momentum equation. Assuming that these conditions are met, find the pressure distribution produced when z is defined vertically upwards. **[10]**

c) Does a stream function exist for the velocity field of part b). If it does, find it and plot and interpret it. **[5]**