NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

JULY 2003 SUPPLEMENTARY EXAMINATION

SMA : 4103 **INTRODUCTION TO FLUID MECHANICS**

TIME 3 HOURS
Answer **ALL** questions in **Part A** and any **THREE** questions in **Part B**.

PART A : Answer **ALL** questions. [40 marks]

1. Let the two-dimensional fluid velocity be given by $v_1 = \dfrac{kx_2x_1}{1+kx_2t}$, $v_2 = 0$. Find

    a) the streamline at time t, passing through the point (α_1, α_2) **[4]**
    b) the pathline for the particle which was at (β_1, β_2) at t=0. **[4]**
    c) the streakline at time t, for the particles which passed through (α_1, α_2) at time $\tau \le t$. **[4]**

2. Let $\mathbf{v}$ be a velocity field for a two-dimensional incompressible fluid. The velocity component v_y at any point is given by

$$v_y = y^2 - x^2$$

Find the v_x-component of velocity. **[4]**

3. Referred to rectangular coordinates (x, y, z), a fluid particle at any time t has the position given by

$$x = a, \quad y = bt^2, z = 0,$$

where a and b are constants. Change to polar cylindrical coordinates and find the velocity field

$$\mathbf{v} = v_r \hat{\mathbf{e}}_r + v_\theta \hat{\mathbf{e}}_\theta + w\hat{\mathbf{e}}_z$$

and its magnitude $|\mathbf{v}|$. **[5:2]**

4. In a two-dimensional incompressible flow the fluid velocity components are given by

$$v_x = x - 4y, \quad v_y = -y - 4x.$$

a) Show that the flow satisfies the continuity equation and obtain the expression for the stream function. **[6]**
b) Calculate the vorticity of the flow. What can be said about the flowfield? **[4]**

5 A source, for a two-dimensional flow, with the strength a and a vortex with the strength b are located at the origin. Determine the equation for velocity potential and the stream function. **[7]**

PART B : Answer any **THREE** questions. Each question carries 20 marks

6 a) Determine the scalar potential function of the vector function
$\mathbf{a} = 2xy\mathbf{i} + x^2\mathbf{j} + 3z\mathbf{k}$ **[6]**

b) A flow field is represented by the potential function

$$\phi = r^{1/2}\cos\frac{\theta}{2}.$$

(i) Find the corresponding stream function **[4]**
(ii) Verify that the flow is incompressible **[4]**
(iii) Calculate the radial and tangential components of acceleration of a particle moving in the flow field. **[6]**

7 a) Determine the vorticity in the following flow fields:
(i) u=kx, v=-ky, w=0
(ii) a forced vortex
(iii) a free vortex
(iv) potential flow around a circular cylinder with its axis normal to the stream direction. **[4:3:3:4]**

b) Use the Bernoulli equation to find the pressure distribution for the flow of a linearly viscous fluid given above in 7a)(i). **[6]**

8 a) Show that $\psi(x,y) = Axy$ satisfies $\nabla^2\psi = 0$ and that the potential function $\phi = A(x^2 - y^2)$ gives the same velocity field. **[6]**

b) A flow field is represented by the potential function

$$\phi = r^{1/2}\cos\frac{\theta}{2}.$$

(i) Find the corresponding stream function **[3]**
(ii) Verify that the flow is incompressible **[3]**
(iii) Calculate the radial and tangential components of acceleration of a particle moving in the flow field. **[8]**

10 a) Explain briefly what is meant by **convective** acceleration,

$$\mathbf{a} = \frac{D\mathbf{v}}{Dt} = \frac{d}{dt}v(x,y,z,t).$$

Represent this in component form. **[5]**

b) Suppose the wall $y = 0$ is fixed, and the rigid wall $y = a$ moves at steady speed V in its plane. Solve the Navier-Stokes equations for the case ρ = constant to show that a possible flow is $v = \frac{Vy}{a}\mathbf{i}$. **[6]**

b) A reservoir has a narrow outlet pipe at the bottom on one side and is full of water(assumed inviscid and incompressible) to height h above the pipe. Let the cross-sectional area of the pipe be a and A(h) be the area of the free surface. Show, stating clearly any assumptions made, that the velocity of the liquid leaving the reservoir is approximately $\sqrt{2gh}$. **[9]**

11 a) A velocity field in cylindrical coordinates is given by

$$\mathbf{v} = \left(\frac{q}{2\pi r} + U\cos\theta\right)\mathbf{e}_r - U\sin\theta\,\mathbf{e}_\theta,$$

where $q = 200 m^2/s$ and $U = 10 m/s$. Show that this is a possible incompressible flow. Locate the stagnation points in the flow. **[5:3]**

b) Consider the velocity field for steady inviscid flow from left to right over a circular cylinder, of radius a, given by

$$\mathbf{v} = U\cos\theta\left[1-\left(\frac{a}{r}\right)^2\right]\mathbf{e}_r - U\sin\theta\left[1+\left(\frac{a}{r}\right)^2\right]\mathbf{e}_\theta.$$

Determine the expressions for the acceleration of a fluid particle moving along the stagnation streamline, $\theta = \pi$, and that along the cylinder surface, $r = a$. Find the points where these accelerations reach maximum and minimum values. **[12]**