NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 4107: TIME SERIES AND SIMULATION

DECEMBER 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.
Throughout this paper the process $\{a_t\}$ is zero mean white noise with variance σ_a^2, that is $a_t \sim N(0, \sigma_a^2)$

SECTION A: Answer ALL questions in this section [55].

A1. The following are monthly incomes($) of small scale business persons in a certain town.

Year	Quarter			
	1	2	3	4
1990	1005	1253	1301	1211
1991	1210	1420	1514	1268
1992	1351	1724	1821	2014
1993	1514	2104	2214	2455

(a) Obtain a time series plot of these incomes and comment. [2]

(b) Obtain centred four point moving averages for the data. [5]

(c) Estimate the trend line of the data. [5]

(d) Obtain seasonal (quarterly) indices of the data. [5]

(e) De-seasonalise the data. [4]

A2. Consider the ARMA(1,2) model

$$Z_t = 0.8Z_{t-1} + a_t + 0.7a_{t-1} + 0.6a_{t-2}$$

Show that

(a) $\rho_k = 0.8\rho_{k-1}$ for $k \geq 3$ and [5]

(b) $\rho_2 = 0.8\rho_1 + \dfrac{0.6\sigma_a^2}{\gamma_0}$ [3]

A3. (a) Determine if the following model is invertible.

$Z_t = a_t - 0.5a_{t-1} + 0.25a_{t-2}$ [4]

(b) Determine if the following models are stationary.

(i) $Z_t = 0.2Z_{t-1} + 0.4Z_{t-2} + a_t$ [3]

(ii) $Z_t - 0.5Z_{t-1} + 0.6Z_{t-2} = a_t$ [3]

A4. Given a U(0,1) random number show how to simulate from;

(a) the geometric distribution; $P(x) = \theta(1-\theta)^{x-1}, \quad x = 1, 2, 3 \ldots;$ [5]

(b) the binomial distribution; Binomial(8, 0.6).[Hint: You can use statistical tables to minimise on calculations] [4]

Generate a geometric (with parameter $\theta = 0.7$) random number and a binomial random number differently using the U(0,1) value of 0.567. [2]

A5. (a) Explain how the generalised rejection method works and its merits over the rejection method. [3]

(b) Show how the probability of rejection is calculated in the generalised rejection method. [2]

SECTION B: Answer THREE questions in this section [45].

B6. Suppose an AR(2) process is given by

$$Z_t = \frac{1}{12}Z_{t-1} + \frac{1}{12}Z_{t-2} + a_t$$

(a) Show that Z_t is a stationary process. [5]

(b) Show (using Yule-Walker equations) that the autocorrelation function of Z_t is given by

$$\rho_k = \frac{45}{77}\left(\frac{1}{3}\right)^{|k|} + \frac{32}{77}\left(\frac{-1}{4}\right)^{|k|}, \text{ for } k = 0, \pm 1, \pm 2, \cdots$$

[10]

B7. (a) The stationary process $\{X_t\}$ has autocovariance function $\gamma_X(k)$. A new process $\{Y_t\}$ is defined by

$$Y_t = X_t - X_{t-1}$$

(i) Show that Y_t is stationary by finding its mean function and its autocovariance function $(\gamma_y(k))$ in terms of $\gamma_x(k)$ [2,3]

(ii) Find $\gamma_y(k)$ when $\gamma_x(k) = \lambda^{|k|}$. [3]

(b) Express the following models in the shift operator notation (in terms of B) and determine if the model is stationary and/or invertible;

(i) $X_t = 0.3X_{t-1} + a_t$ [2]

(ii) $X_t = a_t - 1.3a_{t-1} + 0.4a_{t-2}$ [2]

(iii) $X_t = 0.5X_{t-1} + a_t - 1.3a_{t-1} + 0.4a_{t-2}$ [3]

B8. (a) Show that if U is a $U(0, 1)$ random variable then $N_k = [kU] + 1$ where $[x]$ denotes the integer part of x, has a uniform distribution on the integers $1, 2, 3, \ldots k$. [5]

(b) Obtain the inverse transform method for the Cauchy distribution given by

$$f(x) = \frac{\sigma}{\pi\{\sigma^2 + (x-\mu)^2\}}, \quad -\infty < x < \infty$$

[10]

[Hint: $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$]

B9. Obtain the generalised rejection method for simulating from a half normal distribution using density function $g(x) = \lambda e^{-\lambda x}$, $x > 0$, as the majorising density. Hence determine the value of λ that minimises the probability of rejection. [10]

What is the probability of rejection. [5]

[Hint: for the half normal distribution $f(x) = \sqrt{\frac{2}{\pi}} e^{-\frac{1}{2}x^2}$ $x > 0$]

END OF QUESTION PAPER$_{pp}$