NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 4107

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 4107: TIME SERIES ANALYSIS AND SIMULATION

JANUARY 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [55].

A1. The random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{\alpha\beta^\alpha}{x^{\alpha+1}} & \text{if } x \geq \beta \\ 0 & \textit{otherwise} \end{cases}$$

where both α and β are greater than 0.

(a) Show how to simulate X from a single value of a $U(0,1)$ random variable. [5]

(b) Taking a $U(0,1)$ value of 0.336 and $\alpha = \beta = 1$, generate a random value for X. [3]

A2. Obtain the inverse transform method for the Cauchy distribution (median μ and scale σ) with probability density function given by

$$f(x) = \frac{\sigma}{\pi\{\sigma^2 + (x-\mu)^2\}}$$

[7]

A3. Show that if U is a $U(0,1)$ random variable then $N_k = [kU] + 1$, where [x] denotes the integer part of x, has a uniform distribution on the integers $1, 2, \dots, k$. [5]

A4. Show that if

$$U \sim U(0, 1$$

then

$$a + (b - a)U \sim U(a, b)$$

[5]

A5. For each of the following models:

(a) $X_t = 0.3X_{t-1} + Z_t$

(b) $X_t = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$

(c) $X_t = 0.5X_{t-1} + Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$

where Z_t is a stationay random process (white noise),

express the model in the B notation and determine whether the model is stationary and/or invertible. For equation (a) find the equivalent MA representation. [5,5,5,3]

A6. (a) Find the a.c.f of the second-order MA process given by

$$X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}.$$

[5]

(b) Find the a.c.f ($\rho(k)$) of the first order AR process given by

$$X_t - \mu = 0.7(X_{t-1} - \mu) + Z_t$$

Plot $\rho(k)$, for $k = -6, -5, \dots, 5, 6$. [5,2]

SECTION B: Answer THREE questions in this section [45].

B7. The probability for inter-arrival times of telephone calls at a company's switchboard is exponential with parameter $\lambda = 1$ that is $f(t) = e^{-t}$. We want to simulate a counting process of the number of calls that will be received in some period of time T.

The following algorithm is proposed:

1. set $t = 0$, $I = 0$, T, $S(t) = 0$ (I is a counter of the calls, T is fixed time)
2. generate an inter-arrival time t (use the appropriate algorithm)
3. set $S(t) = S(t) + t$
4. if $S(t) > T$ then stop, otherwise goto 5.
5. set $I = I + 1$ and go to step 2.

(a) Describe the sub-algorithm for step 2. [5]

(b) Explain the purpose of $S(t)$ and I. [3]

(c) Using the following $U(0,1)$ numbers in their given sequence generate 14 inter-arrival times and determine how many events occur within time $T = 6$.
The $U(0,1)$ numbers are: 0.64, 0.17, 0.47, 0.67, 0.89, 0.58, 0.81, 0.74, 0.07, 0.61, 0.14, 0.01, 0.28, 0.28 [7]

B8. (a) Show how the Box-Muller method generates two independent $N(0,1)$ random variables from two $U(0,1)$ random variables. [10]

(b) What are the advantages of the Polar-Marsaglia method over the Box Muller method? [5]

B9. (a) If in a time series data analysis the filters $\{\frac{1}{2}, \frac{1}{2}\}$ and $\{\frac{1}{2}, \frac{1}{2}\}$ are used in series, what is the resultant filter, that is the convolution $\{\frac{1}{2}, \frac{1}{2}\} * \{\frac{1}{2}, \frac{1}{2}\}$? [3]

(b) Explain how autocorrelation is used to derive time series models. [2]

(c) Given a seasonal series of monthly observations $\{X_t\}$, assume that the seasonal factors $\{S_t\}$ are constant so that $S_t = S_{t-12}$ for all t, and that $\{\epsilon_t\}$ is a stationary series of random deviations.

(i) With a global linear trend and additive seasonality, we have
$X_t = a + bt + S_t + \epsilon_t$.
Show that the opartor ∇_{12} acting on X_t reduces the series to stationarity. [3]

(ii) With a global linear trend and multiplicative seasonality, we have
$X_t = (a + bt)S_t + \epsilon_t$.
Does the operator ∇_{12} reduce X_t to stationarity? If not, find a differencing operator which does. [4,3]

B10. (a) Show that the AR(2) process

$$X_t = X_{t-1} + cX_{t-2} + Z_t$$

is stationary provided $-1 < c < 0$. Find the autocorrelation function when $c = -3/16$. [6,3]

(b) Show that the AR(3) process

$$X_t = X_{t-1} + cX_{t-2} + cX_{t-3} + Z_t$$

is non-stationary for all values of c. [6]