NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF CHEMICAL ENGINEERING

AUGUST 2004 SUPPLEMENTARY EXAMINATIONS

SMA4135 FLUID FLOW II

Answer ALL Questions in Section A and THREE Questions in Section B

SECTION A: 28 Marks

1. Define the following terms:
   (a) Laminar flow.
   (b) Steady flow. [**2**; **2**]

2. A flow field is described by the stream function
$$\psi = y - x^2$$
   (a) Sketch the streamlines for $\psi = 0$, $\psi = 1$, and $\psi = 2$.
   (b) Derive an expression for the velocity $\vec{V}$ at any point in the flowfield. [**2**; **2**]

3. The continuity equation when $D\rho/Dt = 0$ is simply $\nabla \cdot \vec{V} = 0$:
   (a) What does it mean exactly and does it make sense.
   (b) Determine if the flow field represented by the velocity components
$$u = x^2y + y^2 \quad and \quad v = x^2 - y^2x$$
   satisfies the continuity equation given above. [**2**; **3**]

4. Given that a fluid flow is described by the velocity field given by
$$\vec{V} = -\alpha(3y\hat{\imath} - x\hat{\jmath}),$$
   find:
   (a) the vorticity,
   (b) the rate of strain tensor,
   (c) the principal rates. [**3**; **3**; **3**]

5. Show that the acceleration at a point in a fluid is given by:
$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}. \quad [6]$$

SECTION B: 72 Marks

6. In unidirectional flow of viscous incompressible fluid the velocity field $u(y,t)$ is found (from the definition of viscousity) to satisfy:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2}, \quad where \quad P = P(x,t)$$

(a) Show that if the flow is steady, then

(i) $\partial P/\partial x = dP/dx$ is constant, and

(ii) $\partial^2 u/\partial y^2 = d^2u/dy^2$ is also constant. [2; 2]

(b) If the flow is steady and $dP/dx = -G$, a constant, show that the flow velocity in the channel with rigid impermeable walls $y = 0,\ h$ is given by

$$u = \frac{Gy}{2\mu}(h-y). \quad [6]$$

(c) Sketch $u(y)$. Is the flow in the right direction? [4]

(d) The plane $y = 0$ is made to oscillate in its own plane so that $u = U\cos(nt)$, where U and n are constants. The fluid occupies the region $y > 0$ and the pressure is uniform. Show that by writing $u = Re[\exp^{int} f(y)]$ the velocity of the fluid is:

$$u = U\exp^{(n/2\nu)^{1/2}y}\cos[nt - (n/2\nu)^{1/2}y]$$

where $\nu = \mu/\rho$. [It may be assumed the flow velocity is bounded by $y \to \infty$]. [10]

7. What are the basic equations of fluid dynamics and to what situations do they apply? In particular, state a specific fluid situation and give a complete set of equations which describe it. [24]

8. It is given that in the motion of a uniform fluid the vorticity $\vec{\omega}$ satisfies

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega}\cdot\nabla)\vec{V} + \nu\nabla^2\vec{\omega} \qquad (1)$$

in the usual notation.

(a) Very briefly explain the physical significance of each term in (1). [6]

(b) The motion is two-dimensional in the (x,y)-plane with stream function ψ. Show that the vorticity can be written $\vec{\omega} = (0, 0, -\nabla^2\psi$. Show that (1) can be written

$$\frac{\partial}{\partial t}\nabla^2\psi - \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)} = \nu\nabla^4\psi. \quad [6] \qquad (2)$$

(c) Given that $\nabla^2\psi = \alpha\psi + \beta y$ where α and β are constants, show that

(i)

$$\nabla^4\psi = \alpha(\alpha\psi + \beta y),$$

(ii)

$$\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \beta\frac{\partial\psi}{\partial x}. \quad [\mathbf{4;4}]$$

(d) Hence show that (2) is satisfied if

$$\alpha\frac{\partial\psi}{\partial t} - \beta\frac{\partial\psi}{\partial x} = \nu\alpha(\alpha\psi + \beta y). \quad [4] \qquad (3)$$

(d) Given that the motion is steady, integrate (3) with respect to x to show that

$$\psi = -\beta y/\alpha + f(y)\exp(-\nu\alpha^2 x/\beta)$$

where $f(y)$ is an arbitrary function.

(e) Given that $\beta/\alpha = -U$, $\nu\alpha^2/\beta = -\lambda$ and $f(y) = -(Ud/2\pi)\sin(2\pi y/d)$. Determine, and sketch, the stream lines $\psi = 0$ when x and y are both small.

9. The steady boundary layer equations in two dimensions are given to be

$$u\frac{\partial u}{\partial x} + v\partial u\partial y = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial x^2}, \qquad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5)$$

in the usual notation.

(a) (i) For what values of the Reynolds number can equations (1) and (2) be regarded as approximate forms of the Navier-Stokes equations?

ii) What term has been neglected in the x-component of the Navier-Stokes equations in the derivation of (1)?

(iii) What information is deduced from the y-component of the Navier-Stokes equations?

(b) In the uniform flow past a semi-infinite plate it is given that the stream function has the form:

$$\psi = (2U\nu x)^{1/2} f(\eta)$$

where $\eta = y(U/2\nu x)^{1/2}$, U is the constant inviscid flow velocity parallel to the plate and the plate coincides with $y = 0$, $x \geq 0$.

(i) Show that $u = Uf'(\eta)$ and $v = -(U\nu/2x)^{1/2}[f - \eta f']$ where dashes indicate differentiation with respect to η.

ii) Explain why (2) is satisfied and show that (1) is satisfied provided $f''' + ff'' = 0$.

(iii) Give the boundary conditions in terms of $f(\eta)$.

(iv) By using this form of solution and comparing the term neglected to a viscou term retained, show that for consistency we require $\nu/Ux << 1$.

10. The equations governing Stokes flow of a uniform incompressible fluid are

$$0 = -\frac{1}{\rho}\nabla P + \nu\nabla^2\vec{u},$$

$$\nabla \cdot \vec{u} = 0,$$

in dimensional variables.

(a) How do these equations differ from the Navier-Stokes equations?

(b) For what values of the Reynolds number can these equations be regarded a approximate forms of the Navier-Stokes equations?

(c) For two-dimensional flow it is given that a solution of the equations can be writte

$$u = \partial\psi/\partial y, \quad v = -\partial\psi/\partial x, \quad p = constant,$$

where p is the pressure. Verify that the continuity equation is satisfied

(d) Show that for two-dimensional motion the equations can be written in the form

$$\nabla^4\psi = 0,$$

where ψ is the stream function.