NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

JULY 2001 SUPPLEMENTARY EXAMINATION

SMA 4234 /TCB3002 VISCOUS FLOW / CHEMICAL PROCESS ENGINEERING

July 2001
3 hours

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Answer <u>all questions</u> in Section A and any <u>three</u> in Section B.

Section A. Answer all questions [40 marks]

1. If ρ the density of a fluid, explain clearly the difference between $D\rho / Dt$ and $\partial\rho / \partial t$. If the flow is incompressible does it imply $\partial\rho / \partial t = 0$ or $D\rho / Dt = 0$? What equation does the velocity vector has to satisfy if the flow is incompressible?

[6 marks]

2. Define the rate-of-strain tensor D_{ij} and give a physical interpretation of it. Calculate D_{ij} for a uniaxial flow defined by $v = (\varepsilon x, - \varepsilon y/ 2, - \varepsilon z /2)$.
Is this an incompressible flow?

3 Define (a) deformation gradient and (b) vorticity tensor. [5 marks]

[3 marks]

4. The drag to a sphere moving steadily at a velocity U through a fluid of viscosity μ is $6 \mu\pi a U$. Describe how this formula can be used to determine the viscosity of a liquid. Is this formula valid for all Reynolds number?

[5 marks]

5. Define Reynolds number.
The force of air resistance to a train moving at 25m/s is to be obtained by the use of a model. The linear dimension of the model is 1/5 that of the train. The model is run in air in a wind tunnel. Find the corresponding air speed in the wind tunnel. Find the relationship between the force measured on the model and that which will act on the train.

[7 marks]

6. When a laminar flow becomes unstable, does this imply turbulence? Describe briefly the transition from laminar to turbulent flows in pipe flow and in flow between two coaxial cylinders.

[6 marks]

7. Show that the boundary-layer equation for a steady two-dimensional jet emerging from an orifice in the form of a long slit is

$$u\,(\partial u/\partial x) + v\,(\partial u/\partial y) = \nu\,(\partial^2 u/\partial y^2)$$

where u,v are the velocity components in the x and y directions respectively and ν is the kinematic viscosity.

Introduce a stream function ψ and look for a similarity solution, i.e. assume

$$\eta = y\,x^{n}, \quad \psi = A\,x^{m} f(\eta)$$

where n,m and A are constants.

Deduce that f satisfies

$$f''' + 2ff'' + 2f'^2 = 0$$

[8 marks]

Section B Answer any 3 questions [60 marks]

8. What is meant by a fluid having a yield stress?
Write down the relationship between the shear stress τ and the shear rate $\dot{\gamma}$ for a Bingham fluid..
Consider the flow of a Bingham fluid in a long circular tube under the action of a constant pressure gradient. Determine the velocity profile and the volumetric flow rate. Compare your results with those of a Newtonian fluid.

[2+3+15marks]

9. The process of wire coating consists of pulling a wire at a constant velocity through a layer of liquid. We can model this process as the flow between two coaxial cylinders with the wire being the inner cylinder. The flow is due to the motion of the inner cylinder and there is no pressure gradient. Deduce the appropriate equations governing the flow assuming that the liquid is an incompressible Newtonian fluid. Determine the flow field, the thickness of the coating and calculate the force acting on the wire.

[8+5+3+4 marks]

10. The flow over a porous wall (y = 0) can be modelled as a flow in the x-direction with a suction in the y-direction. We consider a two-dimensional incompressible flow with velocity distribution (u (y), v (y), 0) and assume that as $y \to \infty$, u and v tend to constants. Deduce from the equation of continuity that v is a constant for all y. Assume that there is no body force and no pressure gradient in the x-direction, write down the equations of motion and the boundary conditions. Hence obtain u.

11 What are the characteristics of turbulent flows?
Discuss the statistical method used in the analysis of turbulent flows.
The equations describing the fully developed turbulent plane channel flow bounded by z = 0 and z = 2h, in the usual notation, are

$$\rho\frac{\partial}{\partial z}\overline{(u'v')} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 U}{\partial z^2}, \qquad 0 = -\frac{\partial P}{\partial y}, \qquad 0 = -\frac{\partial P}{\partial z} - \rho\frac{\partial \overline{w'^2}}{\partial z}$$

The mean flow is in the x-direction.
Use physical arguments to deduce the existence of the viscous, buffer and inertial layers.
Determine the velocity profile in the viscous and inertial layers.

[5+5+10 marks]

The Navier-Stokes equation can be written as

$$\rho(\frac{\partial v_r}{\partial t} + \Sigma v_s \frac{\partial v_r}{\partial x_s}) = -\frac{\partial p}{\partial x_r} + \mu \nabla^2 v_r + \rho b_r$$

The z-component of the equation of motion in (r,θ,z) is

$$\rho(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}) = -\frac{\partial p}{dz} + [\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\tau_{\theta z}) + \frac{\partial}{\partial z}(\tau_{zz})] + \rho b_z$$

END