NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 4236

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

CONTROL THEORY

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40 marks].

A1. (a) Define the term system. [2]

(b) Outline three main features of feedback and open loop control systems. [6]

A2. The linearized equation of motion of a simple pendulum is given by

$$\frac{d^2\theta}{dt^2} + \omega\theta = u.$$

(a) Write the equations of motion in state space form. [5]

(b) Write the transfer function of the system. [5]

A3. Consider the state space form $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ and $y = C\mathbf{x}$. Use the transformation $\dot{\mathbf{x}} = P\mathbf{z}$ to show that the state space can be written as $\dot{\mathbf{z}} = A_1\mathbf{z} + B_1\mathbf{u}$ and $y = C_1\mathbf{z}$. [5]

A4. A system is characterized by the following:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathbf{u}$$

If the forcing function $u(t) = 1$ for $t \geq 0$ and $\mathbf{X}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, find the state of the system at time t using the direct method. [7]

A5. For a system with the characteristic equation $P(s) = s^4 + 2s^3 + 3s^2 + 4s + 5$, determine whether the system is asymptotically stable or not using Hurwitz criteria. [5]

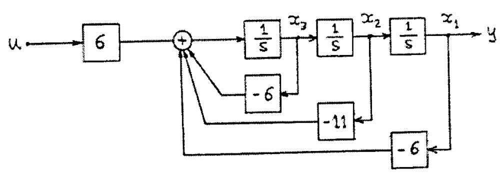
A6. Find the curve $y(x)$ which gives an extreme value of

$$J = \int_0^1 (\dot{y}^2 + 1)dx,$$

when $y(0) = 1$, $y(1) = 2$. [5]

SECTION B: Answer THREE questions in this section [60 marks].

B7. Consider a system given by the block diagram given in the figure below.



(a) Find the state space form of the system. [4]

(b) Construct the Routh array and determine stability of the system. [4]

(c) Using the transformation $\dot{\mathbf{x}} = P\mathbf{z}$ write the state equation as $\dot{\mathbf{z}} = A_1\mathbf{z} + B_1\mathbf{u}$ and $y = C_1\mathbf{z}$, and determine whether the system is controllable and or observable. [12]

B8. (a) Given the generic form for state equations: $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ and $\mathbf{y} = C\mathbf{x} + D\mathbf{u}$,

(i) show that the transfer function matrix mapping the input vector $U(s)$ to the output vector $Y(s)$ is

$$G(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D.$$

[3]

(ii) show that the solution for state equations comprising of the homegenous and forced parts is

$$\mathbf{X}(t) = \Phi(t)\mathbf{X}(0) + \int_0^t \Phi(t-\tau)B\mathbf{U}(\tau)d\tau,$$

where $\Phi(t)$ is the state transition matrix and $\mathbf{X}(0)$ is the initial state vector. [5]

(b) For a system with the system transfer function

$$G(s) = \frac{s+2}{s^3 + 3s^2 + 4s + 6},$$

determine whether the system is observable and or controllable. [12]

B9. (a) Consider the functional I given by

$$I(x) = \int_0^1 \left(\frac{dx(t)}{dt} - 1\right)^2 dt$$

such that $x(0) = 0$ and $x(1) = 1$.

(i) Write the Euler equation for this problem. [2]

(ii) Determine the solution of the Euler equation found in (i) and sketch its graph. [5]

(iii) Argue that the solution of the Euler equation in (ii) indeed minimizes the functional I. [4]

(b) Minimize

$$J = \int_0^2 \frac{1}{2}\dot{x}_2^2 dt$$

subject to $x_2 - \dot{x}_1 = 0$, where $x_1(0) = 1$, $\dot{x}_1(0) = 1$, $x_1(2) = 0$ and $\dot{x}_1(2) = 0$. [9]

B10. (a) If $\dot{x} = u$, $x(0) = 1$, find the optimal control which minimizes

$$J = \int_0^{t_1} (u^2 + x^2)dt$$

when

(i) the final state is prescribed, say $x(t) = x_1$. [5]

(ii) the final state is not prescribed. [5]

(b) Find the extremals of the functional

$$J = \int_0^{\frac{\pi}{2}} (\dot{x}_1^2 + \dot{x}_2^2 + 2x_1x_2)dt$$

subject to the boundary conditions $x_1(0) = 0$, $x_1(\frac{\pi}{2}) = 1$, $x_2(0) = 0$, and also $x_2(\frac{\pi}{2}) = 1$. [10]

B11. (a) Use the Pontryagin's Maximum Principle to completely represent the state solution pair, adjoint solution pair, and the optimal control of the following objective functional

$$\min \quad J(u) = \int_0^1 (x_2 + u^2)dt$$

subject to

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= u \end{aligned}$$

where $x_1(0) = x_2(0) = 0$, $x_1(1) = 1$, and $x_2(1)$ is unspecified. [12]

(b) Using Liapunov criterion, determine stability of the system with the state space form

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -3x_1 - 3x_2 \end{aligned}$$

[8]

END OF QUESTION PAPER