

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY
Faculty of Applied Sciences

SMA 5151

DEPARTMENT OF APPLIED MATHEMATICS

SMA 5151: APPLIED STATISTICS

DECEMBER 2002
Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section (55 Marks)

A1. Show that $S_{xy} = \sum_1^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_1^n (x_i - \bar{x})y_i$. [3]

A2. Consider the sales of a new brand of cars sold at 3 different garages in four different colours: Red(R), White(W), Light Blue(LB) and Black(B). At the first garage the sales were; R=26, W=31, LB=28 and B=30. At the second garage the sales were R=29, W=28, LB=25 and B=31. At the third garage the sales were R=25, W=28, LB=29 and B=32.

- (a) Express this in an appropriate table. [3]
- (b) Construct the ANOVA table suitable for analysing this data. [8]
- (c) Estimate the parameters of the model $Y_{ij} = \mu + \tau_i + e_{ij}$ that is $(\mu, \tau_1, \tau_2, \tau_3$ and $\tau_4)$ for the effects of colour. [10]
- (d) Test the hypothesis that colour has no effect on sales. [5]
- (e) Test the hypothesis that sales depend on the garage. [5]

A3. Describe the one-factor fixed effects model by means of a full rank linear regression model. [5]

- A4. Consider a non-linear regression curve of the form

$$E[Y_i] = \alpha x_i^\beta.$$

The following data is obtained for values of X and Y :

x	1	2	3	4	5
y	12	38	80	170	252

Estimate the regression curve through the use of a logarithmic transformation (estimate the resulting regression line, then transform back to the original model). [10]

- A5. In fitting a curve to some data we normally minimise the sum of squares $\{\sum_{i=1}^n (y_i - \hat{y})^2\}$. A statistician has suggested that we minimise the sum of the absolute values $(\sum_{i=1}^n |y_i - \hat{y}|)$. Does this seem like a reasonable criterion for curve fitting? Does it have advantages? [6]

SECTION B: Answer ANY THREE Questions in this section (45 Marks)

- B6. The marks for an assignment, a test and the final examination for a certain mathematics course are as follows for some 7 students.

Student	1	2	3	4	5	6	7
Assignment(X_1)	50	35	72	45	85	47	79
Test(X_2)	70	64	70	58	89	66	74
Final Exam(Y)	62	45	77	56	91	50	83

- (a) It is assumed that the exam mark depends on the assignment and test marks. State this model in matrix notation. [1]

- (b) If

$$(X'X)^{-1} = \begin{pmatrix} 12.993 & 0.0770 & -0.2480 \\ 0.0770 & 0.0014 & -0.0023 \\ -0.2480 & -0.0023 & 0.0054 \end{pmatrix}$$

find the parameters of the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + c$. [4]

- (c) Calculate the fitted values for the exam and the corresponding residuals for the model in (b) above. Plot the residuals versus the fitted values and comment on them. [10]

- B7. Three competing detergent manufacturers Bright Detergent Co., Pure Detergent Co. and Clean Detergent Co., each claim to be better than the others. An experiment was carried out to verify if they differ in the quality of their detergents. To determine this, each detergent was used to wash up six shirts and the results of brightness measured on a bright-O-meter with higher scores indicating brighter shirts were recorded as follows.

	Bright Detergent	Pure Detergent	Clean Detergent
8	8	7	10
7	6	5	6
9	10	8	6
			3
			5