NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5171 OPERATIONS RESEARCH

NOV/DEC 2002

Answer all questions in section A and any FOUR from section B.

Section A [40 marks] Answer all questions from this section

A1. Prove that for an inventory model where the company produces their own items to stock, the total annual cost of inventory process is as follows:

$$TC = \frac{1}{2}(1-\tfrac{D}{P})QC_h + \frac{D}{Q}C_o$$

Where D = annual demand

C_o = cost of placing an order

C_h = cost of holding one item of stock in inventory for one year

P = annual production rate for the product

and Q = order size **[7marks]**

A2. An investment broker has been given \$1 000 000 to invest. He will choose the investment from a list of 25 stocks. The net return from 10 dollars in stock i is r_i. Given the risks and expected returns for the 25 stocks involved, the following operating policy will be adhered to:

a. No more than \$125 000 will be invested in a single stock.

b. If any amount is invested in a stock, at least \$25 000 will be invested in it.

Formulate as a mixed integer linear programming problem that maximizes return.

[5marks]

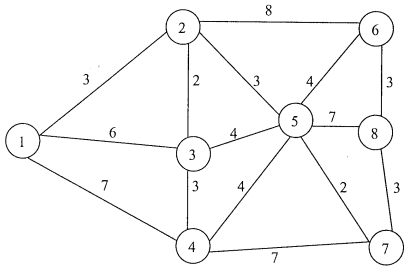
A3. You are given \$10 000 to invest for the next year. The following table gives three possible options, with the net returns in dollars which you can expect if particular states of the economy prevail.

	State of Economy		
	Recession	**Stable**	**Expansion**
Savings Account	400	400	400
Corporate Bonds	100	600	600
Stock Options	-300	300	900

Determine your best decision based on the

(a) Conservative approach
(b) Optimistic approach
(c) Minimax regret approach **[8 marks]**

A4. A large manufacturing plant desires to connect computer terminals at seven locations across the plant site to the computer at its computing center. The network for the terminal locations is shown in the figure below. The values given along the arcs represent the distances in hundreds of metres between the terminal locations, which are denoted by the nodes 2 through 8. The plant manager wishes to determine how to connect all seven terminals with the computer center so that the minimum amount of cable is required. Find the shortest length of cable required.



[8marks]

A5. A company employs service engineers based at various locations through out the country to service and repair their equipment installed in customers' premises. Four requests for service have been received and the company finds that four engineers are available. The distances in kilometers each of the engineers is from the various customers is given in the following table:

		Customers			
		W	X	Y	Z
Engineers	Takura	25	18	23	14
	Tatenda	38	15	53	23
	Tauya	15	17	41	30
	Tinotenda	26	28	36	29

The company wishes to assign engineers to customers to minimize the total distance to be traveled. Formulate as a linear programming model. **[12marks]**

SECTION B: [60 marks]
Answer any FOUR questions from this section

B6. A cargo aeroplane has three compartments for storing cargo: front, center and back. These compartments have capacity limits on both weight and space, as summarized in the table below:

Compartment	Weight capacity (in tones)	Space capacity (in cubic feet)
Front	12	7000
Centre	18	9000
Back	10	5000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain balance of the aeroplane. The following four cargoes (with their unit weight and unit volume specifications) have been offered for shipment on an upcoming flight, as space is available:

Cargo	Weight/unit (tonnes)	Volume/unit (cubic feet)	Profit per unit
1	20	500	320
2	16	700	400
3	25	600	360
4	13	400	290

Each of the cargoes is abundant and any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight. Formulate a linear programming model for this problem. **[15marks]**

B7. A firm, for simplicity, is assumed to produce a single product. The firm has three plants (A,B,C), two warehouses (Y,Z) and four customers (1,2,3,4) geographically dispersed across the country. The firm ships its product from a plant to a warehouse and then to a customer. The information on supplies of this product at various plants, requirements of each customer and cost per unit of shipment from plants to warehouses and from warehouses to customer is provided below as follows:

Warehouse / Plants	Y	Z	Supply at Plant
A	18	23	25
B	19	21	29
C	25	16	16

Thus, total supply = 70.

Customer / Warehouse	1	2	3	4
Y	5	7	14	11
Z	12	15	10	8
Requirement	19	24	17	10

And total requirement = 70.
The firm wants to distribute its product at minimum cost.

(a) Define appropriate variables and formulate the above as a linear programming model.
(b) How is the LP model modified if supply at plant C is raised to 20 units?
(c) How would you modify the linear programming model if the amount shipped from plant B to warehouse Y had to be at least 7 and at most 12? **[15 marks]**

B8. Viddc Chemicals (Pvt) Ltd manufactures three types of chemicals. The company has contracted to supply the following amounts of the three chemicals:

Chemical	**Contracted Sales (kg)**
1	2000
2	3500
3	1800

Viddc Chemicals' production is limited by the availability of processing time in two chemical reactors. Each chemical must be processed first in reactor 1 and then in reactor 2. The following table provides the hours of processing time available next month for each reactor and the processing time required in each reactor per kg of each chemical:

Reactor Processing Times (hour/kilogram)				
	Chemical			Reactor
	1	2	3	Availabilities
Reactor 1	0.05	0.04	0.01	200 hours
Reactor 2	0.02	0.06	0.03	150 hours

Owing to the limited availability of reactor processing time Viddc Chemicals has insufficient capacity to meet its demand with in-house production. Consequently Viddc Chemicals must purchase some chemicals from vendors having excess capacity and resell them to its own customers. The following table provides each chemical's in-house production cost and outside purchase cost:

Chemical	In-House Production Cost ($/kg)	Outside Purchase Cost ($/kg)
1	2.50	2.80
2	1.75	2.50
3	2.90	3.25

Viddc Chemicals's objective is to fill its customers' orders with the cheapest combination of in-house production and outside purchases. In short the company must decide how much of each chemical to produce in-house and how much of each chemical to purchase outside.

(a) Let m_i = quantity of chemical i manufactured by the company. i = 1,2,3 and b_i = quantity purchased of chemical i , i = 1,2,3. Formulate as a linear programming problem

(b) Use the computer output attached in appendix A to answer the following questions. In some parts, the computer output may provide insufficient information to allow you to answer in which case you should state, 'Insufficient Information'.

(i) Viddc Chemicals anticipates an increase in inhouse production cost per kilogram of chemical 1 from $2.50 to $2.54. If this occurs, what are the optimal values of the decision variables and what is the optimal objective function value?

(ii) Viddc Chemicals has contracted a new vendor to supply with chemical 2 at a cost of $2.40 per kilogram. With this new purchase cost, what are the optimal values of the decision variables and what is the optimal objective function value?

(iii) Suppose that the contracted sales of chemical 1 is exactly 2300kg. What is the optimal objective function value?

(iv) Suppose that the contracted sales of chemical 1 is exactly 1900kg. What is the optimal objective function value?

(v) If the availability of reactor 2 increases from 150 hours to 170 hours, what is the optimal total cost?

(vi) If the availability of reactor 2 decreases from 150 hours to 140 hours, what is the optimal total cost? **[15 marks]**

B9. Tigamuchire Investors of Bulawayo recently purchased land and is attempting to determine the size of a holiday resort it should build. The sizes of holiday resorts are being considered: small, medium, and large. At the same time, an uncertain economy makes it difficult to ascertain the demand for the new resort area. Tigamuchire's management realizes that a large development followed by a low demand could be very costly to the company. However, if Tigamuchire makes a conservative small development decision and then finds a high demand, the firm's profits will be lower than they might have been. With the three levels of demand, low, medium and high, Tigamuchire's management has prepared the following profit ($1000s) payoff table.

Decision	**Demand**		
Alternatives	Low (s_1)	Medium (s_2)	High (s_3)
Small (d_1)	400	400	400
Medium (d_2)	100	600	600
Large (d_3)	-300	300	900

The probabilities of each of the states of nature are:
P (High demand) = 0.45
P (Medium demand) = 0.35
P (Low demand) = 0.20
(a) Construct a decision tree for this problem. What is the recommended decision using the expected value approach?

(b) What is the expected value of perfect information (EVPI) for this problem?
(c) The resort company is conducting a survey that will help evaluate the demand for the development. The survey will result in three indicators of demand: weak (I_1), average (I_2) or strong (I_3). The conditional probabilities are shown in the table below:

	$P(I_k/s_k)$		
	I_1	I_2	I_3
s_1	0.6	0.3	0.1
s_2	0.4	0.4	0.2
s_3	0.1	0.4	0.5

(i) What is the resort company's optimal strategy?
(ii) What is the value of survey information?
(iii) What is the efficiency of the survey information? **[15marks]**

B10. The Midtown City Council is reviewing housing proposals for a new development area. There is some dispute among various interest groups as to what goals should be sought. The zoning Committee has recommended three types of housing: one-family houses, deluxe condominiums and apartments. There are 50 acres available for zoning. The Zoning Committee has compiled the following data for each type of housing:

	Housing Type		
	One-family	Deluxe condominiums	Apartment
Land Usage (acres per unit)	0.25	0.20	0.125
Families Housed per unit	1	4	6
Tax Base Generated per Unit	$50 000	$100 000	$150 000
Taxes Required for City Services per Unit	$4 000	$8 000	$10 000

The City council hired a public-opinion survey company to asses the priorities of the citizens. Using this survey, the City Council has established the following goals (in decreasing order of priority):

1. Provide housing for at least 500 families
2. Establish at least $5 000 000 worth of new tax base
3. Taxes for city services should be limited to $250 000

4. Reserve at least 5 acres for a neighbourhood park area

(a) Formulate a goal programming model the City Council can use to decide how to zone the new development area.
(b) What modifications to your goal program are necessary if the second goal were stated as "Maximize the worth of the new tax base"?
(c) What modifications to your goal program are necessary if a fifth goal is added stating, "If the first goal is not achieved, the amount of underachievement should not exceed 25 families"? **[15marks]**

B11. (a) Assume that a production line operates such that the production lot size model is applicable. Given that annual demand is 6400 units, cost of setting up production run is $100 and holding cost per unit is $2 per annum. Compute the minimum production lot size for each of the following production rates

(i) 8000 units per annum
(ii) 10 000 units per annum
(iii) 32 000 units per annum
(iv) 100 000 units per annum

Compute the economic order quantity (EOQ). What two observations can you make about the relationship between the EOQ and the economic production lot size?
(b) The daily demand for an item during a single period occurs instantaneously at the start of the period. The probability density function of the demand is uniform between 0 and 10 units. The unit holding cost of the item during the period is $0.50. A fixed cost of $25 is incurred each time an order is placed. Determine the optimal inventory policy for the item. **[15marks]**

END OF EXAMINATION

APPENDIX A

*** OPTIMUM SOLUTION SUMMARY ***

Title: Sable Chemicals
Final iteration No.: 6
Objective value (min) =18225.0000

Variable	Value	Obj Coeff	Obj Val Contrib
x1 m1	2000.0000	2.5000	5000.0000
x2 m2	1833.3334	1.7500	3208.3335
x3 m3	0.0000	2.9000	0.0000
x4 b1	0.0000	2.8000	0.0000
x5 b2	1666.6665	2.5000	4166.6660
x6 b3	1800.0000	3.2500	5850.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	2000.0000	0.0000
2 (=)	3500.0000	0.0000
3 (=)	1800.0000	0.0000
4 (<)	200.0000	26.6667-
5 (<)	150.0000	0.0000-

*** SENSITIVITY ANALYSIS ***

Objective coefficients -- Single Changes:

Variable	Current Coeff	Min Coeff	Max Coeff	Reduced Cost
x1 m1	2.5000	-infinity	2.5500	0.0000
x2 m2	1.7500	1.6000	1.8000	0.0000
x3 m3	2.9000	2.8750	infinity	-0.0250
x4 b1	2.8000	2.7500	infinity	-0.0500
x5 b2	2.5000	2.4500	2.6500	0.0000
x6 b3	3.2500	-infinity	3.2750	0.0000

Right-hand Side -- Single Changes:

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (=)	2000.0000	0.0000	2727.2726	2.7500
2 (=)	3500.0000	1833.3335	infinity	2.5000
3 (=)	1800.0000	0.0000	infinity	3.2500
4 (<)	200.0000	173.3333	infinity	0.0000
5 (<)	150.0000	40.0000	190.0000	-12.5000

Objective Coefficients -- Simultaneous Changes d:

Nonbasic Var	Optimality Condition
x3 m3	-0.0250 + -0.5000 d5 + 1.0000 d6 + 0.5000 d2 - d3 <= 0

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 1        -0.0500 +      1.0000 d1 +      0.3333 d5 +      -0.3333 d2
             - d4 <= 0
.11       -12.5000 +    -16.6667 d5 +     16.6667 d2 <= 0

Right-hand Side Ranging -- Simultaneous Changes D:
------------------------------------------------------------------------
Basic Var    Value/Feasibilty Condition
------------------------------------------------------------------------
x1 m1      2000.0000 +     1.0000 D1 >= 0
x5 b2      1666.6665 +     0.3333 D1 +     1.0000 D2 +    -16.6667 D5
              >= 0
x6 b3      1800.0000 +     1.0000 D3 >= 0
sx10          26.6667 +   -0.0367 D1 +     1.0000 D4 +     -0.6667 D5
              >= 0
x2 m2      1833.3334 +    -0.3333 D1 +    16.6667 D5 >= 0
End of Solution Summary
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