

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

SMA 5171: Operations Research

December 2004

Time: 3 hours

Answer ALL questions from Section A and any THREE questions from Section B

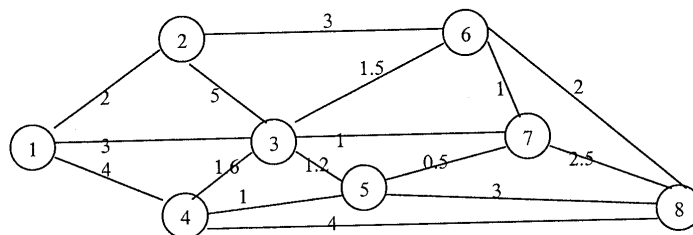
- 1 Briefly explain the importance of each of the following in operations research:
- (a) Multi-objective Programming;
  - (b) Decision Analysis;
  - (c) Linear programming;
  - (d) Economic Order Quantity;
  - (e) Shortest Route algorithm.

[10 marks]

- 2 Considering a single item that is manufactured by a company and has a deterministic demand of  $D$  items per annum. Let  $Q$  be the quantity produced on each run,  $C_h$  is the holding cost per item, and  $C_o$  is the cost of setting a production run, and  $P$  the annual rate at which items can be produced. Derive the Economic Production Lot Size formula and show that when this is used, annual production set-up costs are equal to annual holding costs.

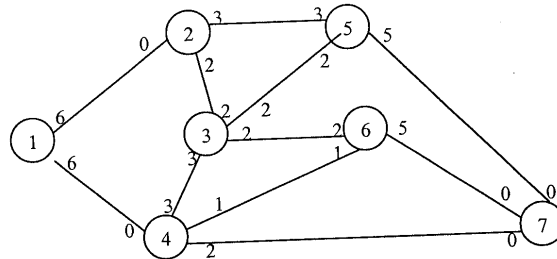
[6 Marks]

- 3 NUST is installing a computerized electronic mail system that will enable messages to be transmitted instantly among eight departments. The network with possible electronic connections among the departments is shown below. Distances between departments are shown (in hundreds of meters). Develop a design for the department communication system that will minimise the total length of connections among the eight departments. Also calculate the minimum length.



[6 marks]

- 4 Comoil company owns a pipe line network that is used to transmit oil from its source to several garages. The network is as below.



- (a) How long will it take to satisfy garage 7 demand of 100 000 litres?  
 (b) If a break occurs on line 2-3 and is closed down, what is the maximal flow for the system? [4+3 Marks]

- 5 The XYZ company purchases a component used in the manufacture of automobile generators directly from the supplier. XYZ's generator production operation, which is operated at a constant rate, will require 1000 components per month throughout the year (12000 units per year). Assume that the ordering costs are \$25 per order, the unit cost is \$2.50 per component, and that the annual holding costs are charged at 20%. There are 250 working days per year and the lead time is 5 days.

- (a) Derive the economic order quantity formula.  
 (b) What is the EOQ for this component?  
 (c) What is the reorder point?

[5+3+3 marks]

**Section B**  
**Answer any 3 questions from this section**

- 6 Amakhosi Productions is considering producing a comedy series "Amakorokoza" for ZBC TV. While ZBC may reject the comedy, it may also purchase the program for 1 or 2 years. Amakhosi may decide to produce the series or transfer the rights to a competitor for \$100 000. Amakhosi's profits are summarised in the following profit (\$1000s) payoff table:

	States of Nature		
	Reject	1 year	2 Years
$d_1$ (Produce comedy)	-100	50	150
$d_2$ (Sell to competitor)	100	100	100

It is assumed that the probability estimates for the states of nature are  $P(\text{reject}) = 0.2$ ,  $P(1 \text{ year}) = 0.3$ , and  $P(2 \text{ years}) = 0.5$ .

- (a) What should Amakhosi Productions do?  
 (b) What is the maximum that Amakhosi should be willing to pay for inside information on what ZBC will do?  
 (c) The payoff table (profit in \$1000s) for Amakhosi Productions is as follows:

Decision Alternatives		States of Nature		
		$s_1$	$s_2$	$s_3$
Produce comedy	$d_1$	-100	50	150
Sell to competitor	$d_2$	100	100	100
<b>Probability of states of nature</b>		0.2	0.3	0.5

For a consulting fee of \$2500, an agency will review the plans of the comedy and indicate the overall chances of a favorable ZBC reaction to the series. If the special agency review results in a favorable ( $I_1$ ) or unfavorable ( $I_2$ ) evaluation, and assume that Amakhosi believes that the following conditional probabilities are realistic appraisals of the agency's evaluation accuracy:

$$\begin{aligned}
 P(I_1 / s_1) &= 0.3 & P(I_2 / s_1) &= 0.7 \\
 P(I_1 / s_2) &= 0.6 & P(I_2 / s_2) &= 0.4 \\
 P(I_1 / s_3) &= 0.9 & P(I_2 / s_3) &= 0.1.
 \end{aligned}$$

- (i) Show the decision tree for this problem.  
 (ii) What is the recommended decision strategy and the expected value, assuming that the agency information is obtained?

**[5+3+7+5 marks]**

- 7 Comp enterprises manufactures the Central Processing Unit (CPU) for a line of personal computers. The CPU are manufactured in Seattle, Columbus and New York, and shipped to warehouses in Pittsburgh, Mobile, Los Angeles and Washington DC for further distribution. The transportation table below shows the number of CPUs available at each plant and the number of CPUs required by each warehouse. The shipping costs (dollars per unit) are also shown in each cell.

Plant	Warehouses					CPUs Available
	Pittsburgh	Mobile	Denver	Los Angeles	Washington	
Seattle	10	20	5	9	10	9000
Columbus	2	10	8	30	6	4000
New York	1	20	7	10	4	8000
	3000	5000	4000	6000	3000	21000

- (a) Develop a network representation of the problem [2marks]  
 (b) Determine the amount that should be shipped from each plant to each warehouse to minimise the total shipping costs [8marks]  
 (c) Consider the all integer linear program

$$\begin{aligned} \text{Max } z &= 8x_1 + 5x_2 \\ \text{subject to} \\ x_1 + x_2 &\leq 6 \\ 9x_1 + 5x_2 &\leq 45 \\ x_1, x_2 &\geq 0 \text{ and integer} \end{aligned}$$

Use the branch and bound procedure to solve this problem. [10 marks]

- 8 Amvest (Pvt) Ltd has \$500 000 available for investment and seeks to determine how much of this amount is to be invested in each of four available investment alternatives: stock X, stock Y, bond X and bond Y. A maximum of \$105 000 may be invested in bonds of type X and a maximum of \$100 000 may be invested in bonds of type Y. Annual net return is listed in the table below:

Annual net Return on Investment	
Stock X	20%
Stock Y	10%
Bond X	9%
Bond Y	11%

Amvest is aware of the fact that there is a considerable amount of risk associated with investment in stock X. Therefore Amvest has determined that it will not invest an amount in stock X that exceeds one-fourth of its total investment. Also the total amount invested in stock Y must be at least three times the amount invested in stock X. Additionally, Amvest requires that its investment in bonds must be at least as great as one-half its investment in stocks. Management wants to maximise return and thus the total amount of money available after an investment period of one year. Formulate this problem as a linear programming problem.

[20 marks]

- 9 Mazda Motors (MM) has just introduced a new luxury touring sedan. As part of its promotional campaign, the marketing department has decided to send personalised invitations to test drive the new sedan to two target groups: (1) current owners of an MM luxury automobile and (2) owners of luxury cars manufactured by one of MM's competitors. The cost of sending a personalized invitation to each customer is estimated to be \$1 per letter. Based on previous experience with this type of advertising, MM estimates that 25% of the customers contacted from group 1 and 10% of the customers contacted from group 2 will test drive the new sedan. As part of this campaign, MM has set the following goals:

Goal 1: Get at least 10 000 customers from group 1 to test drive the new sedan.

Goal 2: Get at least 5 000 from group 2 to test drive the new sedan

Goal 3: Limit the expenses for sending out the invitations to at most \$70 000.

Assume that goals 1 and 2 are P1 priority level goals and that goal 3 is a P2 priority level goal.

- (a) Suppose goals 1 and 2 are equally important, formulate a goal programming model of the MM problem.
- (b) Explain fully how the computer procedure is used to solve the model formulated in part (a).
- (c) If management believes that contacting customers from group 2 is twice as important as contacting customers from group 1 What should MM do?

[10+6+4marks]

**END OF QUESTION PAPER**