**NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY**

FACULTY OF APPLIED SCIENCE

APPLIED MATHEMATICS DEPARTMENT

MSc IN OPERATIONS RESEARCH

# OPERATIONS MANAGEMENT
## - SMA 5172 -

DATE: **AUGUST 2004** **Supplementary Examination**

DURATION: **3 HOURS**

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**INSTRUCTIONS:**

Each question carries **25 marks**.

The **CANDIDATE** is required to answer **ALL**.

This question paper has four questions and six printed pages.

## QUESTION 1

In order to complete the wing assembly of an experimental aircraft, Scott De Witte has laid out the major steps and seven activities involved. These activities have been labelled A through G in the following table, which also shows their estimated completion times and immediate predecessors.

| ACTIVITY | ‘a | m | b | IMMEDIATE PREDECESSORS |
|---|---|---|---|---|
| A | 1 | 2 | 2 | - |
| B | 2 | 3 | 3 | - |
| C | 4 | 5 | 6 | A |
| D | 8 | 9 | 11 | B |
| E | 2 | 5 | 5 | C,D |
| F | 3 | 5 | 6 | B |
| G | 1 | 2 | 3 | E |

a) Determine the expected time and variance for each activity. (8)

b) Draw the PERT diagram for the Scott De Witte project. (5)

c) Determine the critical path and the expected completion time. (5)

d) What is the project variance? (2)

e) What is the probability that the job can be completed in 20 days? (5)

## QUESTION 2

The operations of a sporting goods company makes skis in three plants throughout the world. The plants supply four company-owned warehouses that distribute the skis directly to the ski shops. Depending on which mode is cheaper, the product is air-freighted or trucked from the plants to the warehouses. The monthly capacities of the plants in terms of the number of skis that can be made are

| PLANT | CAPACITY |
|---|---|
| Juarez | 100 |
| Seoul | 300 |
| Tel Aviv | 200 |

And warehouse demand requirements for the next month are

| WAREHOUSE | DEMAND |
|---|---|
| Frankfurt | 150 |
| New York | 100 |
| Phoenix | 200 |
| Yokohama | 150 |

The table below shows the various point- to-point costs of shipping a pair of skis.

| FROM PLANT | TO WAREHOUSE | | | |
|---|---|---|---|---|
| | Frankfurt | New York | Phoenix | Yokohama |
| Juarez | 19 | 7 | 3 | 21 |
| Seoul | 15 | 21 | 18 | 6 |
| Tel Aviv | 11 | 14 | 15 | 22 |

a) Derive the objective function for minimising the total shipping cost (12)

b) Derive the three plants capacity constraints for this problem. (5)

c) Derive the four Warehouse demand constraints for this problem. (6)

d) Write the Non-negativity conditions. (2)

## QUESTION 3

Given that:

k= Fixed cost per order

A = Annual number of items demanded

c = Unit cost of procuring an item

h= Annual cost per dollar value of holding items in inventory

T = Time between orders

And Q = Order quantity

a) Show that for the simple economic order quantity inventory system model,

Total Annual Relevant Cost, TC(Q) = (A/Q)k + hc(Q/2) (5)

b) Derive the formula for the Optimal Order Quantity, Q*. ($2^1/_2$)

c) Draw the graphical representation of the inventory cost components. (5)

d) Show that if $Q_s^*$ is economic order quantity for the simple order quantity model, and $Q_b^*$ is the optimal order quantity for the model with backordering, where p is the shortage penalty,

$$\frac{Q_b^*}{Q_s^*} = \sqrt{\frac{p + hc}{p}}$$

($12^1/_2$)

## QUESTION 4

A stationery store stocks printer cartridges. The demand and lead-time are certain, but historical experience indicates that the lead-time demand distribution in **Table 3** below applies. The mean annual demand is **5 000** items. Given that the following parameters apply:

| | | |
|---|---|---|
| **K** | = | \$20 per order |
| **C** | = | \$0.45 per item |
| **H** | = | \$0.12 per dollar value per year |
| **$p_S$** | = | \$0.10 per item short |
| **$p_R$** | = | \$0.90 per item. |

The customers are willing to a slight delay in getting orders fulfilled, otherwise they will gc elsewhere to purchase their floppies stock-outs.

| **Possible Demand d** | **Probability Prob(D=d)** |
|---|---|
| 90 | 0.05 |
| 100 | 0.12 |
| 110 | 0.17 |
| 120 | 0.22 |
| 130 | 0.19 |
| 140 | 0.14 |
| 150 | 0.05 |
| 160 | 0.03 |
| 170 | 0.02 |
| 180 | 0.01 |

(i) Find the optimal re-order quantity $r^*$ and the optimal quantity $Q^*$.

(ii) Determine the total annual expected cost of this policy (TEC).

Given that:

$$TEC(r,Q) = \left[\frac{A'}{Q}\right]K + hc\left[\frac{Q}{2} + r - \mu + b(r)\right] + (p_S + p_R - c)\left[\frac{A'}{Q}\right]B(r)$$

where $A' = A\left(1 - {}^{B(r)}\!/_{Q}\right)$

$$\Pr_L\left[D \le r^*\right] \ge \frac{(p_S + p_R - c)A}{hcQ + (p_S + p_R - c)A} \quad \text{(with lost sales)}$$

$$Q^* = \sqrt{\frac{2A\left[k + (p_S + p_R - c)B(r)\right]}{hc}} \quad \text{(with lost sales)}$$

where **B(r)** is the expected number of items short per inventory.

# THE END