NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTYOF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

MSc IN OPERATIONS RESEARCH

OPERATIONS MANAGEMENT – SMA5172

1ST SEMESTER EXAMINATIONS DECEMBER 2004

Time allowed: 3 hours
Answer FOUR questions, each question carries 25 marks

QUESTION 1

a) Given that the makespan lower bound based on machine j is

$$LB_j = \min_i \left\{ \sum_{r=1}^{j-1} p_{ir} \right\} + \sum_{i=1}^{N} p_{ij} + \min_i \left\{ \sum_{r=j+1}^{M} p_{ir} \right\}, where$$

p_{ij} is the processing time of job i on machine j. $\{ i=1,\ldots,N \text{ and } j=1,\ldots,M\}$.

Determine the lower bound of the following four-machine flow shop problem.

	Processing Machine Times			
Job	1	2	3	4
1	2	3	1	5
2	4	2	6	9
3	1	4	8	5
4	6	3	5	5
5	4	2	3	7

b) Using Campbell et. al.'s Modified Johnson algorithm determine the M-1 minimum possible makespan sequences for this problem.

c) What is the optimal sequence? And what is the optimal makespan?

QUESTION 2

Given that:

k = Fixed cost per order

A = Annual number of items ordered

c = Unit cost of procuring an item

h = Annual Cost per dollar value of holding items in inventory

T = Time between orders

Q = Order quantity without backordering

Q_b= Order quantity with backordering

p = Shortage penalty

and, S = Stock on hand at beginning of cycle

a) Show that for an inventory policy with backordering

$$Tc(Q_b, S) = k\left(\frac{A}{Q_b}\right) + hc\left(\frac{S^2}{2Q_b}\right) + p\frac{(Q_b - S)^2}{2Q_b}.$$

[8]

b) Show that, if Q* is the optimal economic order quantity for the simple economic order quantity model without backordering

$$S^* < Q^* < Q_b.$$

[8]

c) Show that

$$MaximumShortage = Q^*\left\{\left(1+\left(\frac{hc}{p}\right)\right)^{1/2} - \left(1+\left(\frac{hc}{p}\right)\right)^{-1/2}\right\}.$$

[4]

d) If k=\$100, A= 1000, c=\$20, h=\$0.20, p=\$3.65, Calculate

i) Q^*

ii) S^*

iii) Q_b^*

iv) *Maximum Shortage.*

[5]

QUESTION 3

a) Show the four possible ways of correctly representing two concurrent activities A and B in a CPM network. [2]

b) General Foundry Ltd., a metalworks plant, has long been trying to avoid the expense of installing air pollution control equipment. The local environmental authority has recently given them 16 weeks to install a complex air filter system on its main smokestack. General Foundry was warned that it would be forced to close unless the device is installed in the allotted period. The following are the details of the project:

Activity	Description	Immediate Predecessors	Variance
A	Build internal components	-	4/36
B	Modify roof and floor	-	6/36
C	Construct collection stack	A	4/36
D	Pour concrete and install frame	B	5/36
E	Build high-temperature burners	C	36/36
F	Install control system	C	20/36
G	Install air pollution device	D,E	64/36
H	Inspection and Testing	F,G	4/36

i) Draw the network diagram. [5]

ii) Given that the critical path is A-C-E-G-H, and the expected project completion time is 15 weeks, what is the probability that the project will be completed in 16 weeks? [5]

c) Given the following activity costs for General Foundry, and that management wants the job to be completed in 12 weeks to reduce the risk of being forced to close, derive the linear program that can be used to determine the minimum cost-crashing schedule. [9]

	Time In Weeks		'000 $ Cost	
Activity	Normal	Crash	Normal	Crash
A	2	1	22	23
B	3	1	30	34
C	2	1	26	27
D	4	3	48	49
E	4	2	56	58
F	3	2	30	30.5
G	5	2	80	86
H	2	1	16	19

d) Determine the minimum cost crashing schedule and the new project cost. [4]

QUESTION 4

a) Explain the meaning of the objective function and constraints of the mathematical model below given that they describe a mixed-model-line-sequencing problem.

Let X_{jn} = (1 if item j is placed in the nth position and 0 otherwise),

and $j(n)$, denote the type of item placed on the nth position.

$$Minimise \max_{1 \le n \le N} \sum_{j=1}^{n} \left| \sum_{i \in S_{kb}} t_{i,j(n)} - nC_{kb} \right|$$

Subject to

$$\sum_{n=1}^{N} X_{jn} = N_j \qquad j = 1,\dots,P \qquad (1)$$

$$\frac{n.N_j}{N} - s_1 \le \sum_{n=1}^{n} X_{jn} \le \frac{n.N_j}{N} + s_1 \quad n = 1,\dots,N, \ \ j = 1,\dots,P \qquad (2)$$

$$\sum_{n=1}^{N} \sum_{j=1}^{P} \sum_{i \in S_{kb}} t_{ij} X_{jn} \le (n + s_2) C_k \qquad (3)$$

[8]

b) Given that a mixed model line is to run four products simultaneously and that the bottleneck workstation is assigned a workload of 68 seconds per cycle, determine the product quantities per cycle and the best sequence of release of orders into production. [17]

Model	**Sales**	**Station Time (s)**
Red Z	250	72
Blue Q	250	68
Black R	500	68
RWB	500	66

QUESTION 5

a) State the mathematical model for a knapsack problem. Explain the meaning of both the objective function and the constraint/s. [5]

b) Eight parts are being considered for an automated workcell. These parts are currently purchased from a vendor. The workcell will be available for 250 hours per period. The cell is charged at the rate of $50 per hour, which includes all related expenses.

Use a greedy knapsack heuristic to determine which parts the new cell using the data below should produce.

What are the savings? [17]

PART TYPE

	1	2	3	4	5	6	7	8
Unit Purchase Price	200	155	300	125	300	86	93	165
Material Cost	45	35	124	50	120	34	36	114
Demand per Period	100	50	50	75	60	30'	50	600
Hours per Unit	1	2	4	1	2	1	1	0.5

c) Why is this make-or-by decision considered as a knapsack problem? [3]

QUESTION 6

A three-machine cell has four part types to produce in the next period. Machine A costs $50 per hour whereas B and C are charged at the rate of $30 per hour. Any machine can be used to produce any part: however, machine A has better precision, which is important for part type b. Demand for parts a, b, c and d is (20,40,40,30), respectively. Each machine can hold three tools and has 60 hours available. Tools will be replaced as they wear out, and so you need not consider this information.

Given the data in the table below, formulate the loading problem as a mathematical program with the objective of minimising production costs. [25]

Part	Operation	Machine A	Machine B	Machine C	Tool Number
a	1	0.2	0.3	0.3	3
	2	0.4	0.6	0.7	2
	3	0.1	0.1	0.1	1
b	1	0.5	0.6	0.7	1
	2	0.3	0.5	0.5	2
c	1	0.2	0.2	0.3	3
d	1	0.3	0.4	0.4	1
	2	0.1	0.2	0.2	2