NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA5252

DEPARTMENT OF APPLIED MATHEMATICS

SMA5252 INDUSTRIAL STATISTICS

MAY 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B. A table of constants used for constructing quality control charts is given in the APPENDIX at the end of the question paper.

SECTION A: Answer ALL questions in this section [40].

A1. The incomplete table below displays the signs of the coefficients in the contrasts of the main and interaction effects in a 2^3 factorial design.

(a) Complete the table. [2]

(b) (i) Construct a 2^3 factorial design with ABC confounded with blocks.

(ii) Construct a skeleton ANOVA table for your design in (i) when it is replicated r times.

(iii) In practice which effects in a 2^k factorial design should be confounded with blocks and why?

[4+4+1]

(c) (i) Construct a 2^{3-2} fractional factorial design with generators AC and BC.

(ii) Construct a skeleton ANOVA table for your design in (i) when it is replicated r times.

(iii) In practice what is the criterion of choosing the generators of a fractional 2^k factorial design?

[4+2+1]

Total	Factorial effect							
	I	A	B	C	AB	AC	BC	ABC
(1)	+	−	−	−				
a	+	+	−	−				
b	+	−	+	−				
c	+	−	−	+				
ab	+	+	+	−				
ac	+	+	−	+				
bc	+	−	+	+				
abc	+	+	+	+				

A2. Suppose that $Z_1, Z_2, ..., Z_n$ are independent observations on a product quality characteristic when the production process is in statistical control. Furthermore, suppose that $U = U(Z_1, Z_2, ..., Z_n)$ is the sample statistic that summarises the quality characteristic measurements, and that the distribution of U has mean μ and variance σ^2. An unbiased estimator of σ^2 is given by $\widehat{\sigma}^2 = S(Z_1, Z_2, ..., Z_n)$. If μ and σ^2 are known, then a typical control chart for this production process is given by:

$$\mu \pm l\sigma$$

where l is a positive real number.

(a) If μ and σ are known, explain how you can determine the α probability limits of the control chart from the distribution of U.

(b) Explain the equivalence of using the control chart in (a) and hypotheses testing about the state of the production process.

(c) If U is the sample mean and if n is large what value of l gives the 0.05 probability limits of the control chart and why?

(d) If μ and σ are unknown, explain how you can estimate them from k independent samples of size n taken from the production process when it is in statistical control.

[3+4+2+4]

A3. Consider a $k - Out - of - n$ system with independent components. Suppose that the i^{th} component functions with probability p_i.

(a) Find the reliability function of the $2 - Out - of - 3$ system.

(b) The series and parallel systems are both special cases of a $k - Out - of - n$ system. Explain. Also, find the reliability functions of the two systems when $p_i = p$ for all $i = 1, 2, ..., n$.

[3+6]

SECTION B: Answer THREE questions in this section [60].

B4. Suppose that a parallel system has two components, and that the components function independently of each other for an amount of time (in hours) exponentially distributed with mean $\frac{1}{\theta}$. Find the system's (*a*) reliability function, (*b*) failure rate function and (*c*) expected life.

Hint: The probability density function of an exponential distribution with mean $\frac{1}{\theta}$ is given by:

$$f(t) = \begin{cases} \theta e^{-\theta t} & t > 0,\ \theta > 0, \\ 0 & \text{otherwise.} \end{cases}$$

B5. A company is interested in comparing two different sunscreens S_1 and S_2 for protecting the skin of light skinned people. A random sample of 10 females aged 20-25 participated in the study. For each person two 2 cm x 2 cm squares were marked off on each cheeck. Sunscreen S_1 was randomly assigned to two squares and S_2 assigned to the remaining two squares. A reading based on the colour of skin in a square was made prior to the application of a fixed amount of the assigned sunscreen, and then after application and exposure to the sun for a two-hour period. The data recorded were differences (postexposure minus preexposure) for the persons in the study.

(a) The design of the experiment is (*i*) a randomised block design and / or (*ii*) a repeated measures design. Why? [3+1]

(b) The appropriate linear statistical model for the experiment is a mixed effects model. Why? [3]

(c) The ANOVA of the data gave the following incomplete ANOVA table.

Source	SS	df	MS	F
Person	517.49	....	....	....
Sunscreen	4.49	....	....	....
Person × Sunscreen	5.97	....	....	....
Error	....	....	....	
Total	530.59	....		

For a two-factor factorial experiment with factor A (with a levels) fixed and factor B (with b levels) random we have (in our usual notation):

$$\begin{aligned} E[MSA] &= \sigma^2 + r\sigma^2_{\alpha\beta} + \frac{rb}{a-1}\sum_{i=1}^{a}\alpha_i^2 \\ E[MSB] &= \sigma^2 + ar\sigma^2_{\beta} \\ E[MSAB] &= \sigma^2 + \sigma^2_{\alpha\beta} \\ E[MSE] &= \sigma^2 \end{aligned}$$

where r is the number of replications of each cell.

(i) Complete the above ANOVA table.

(ii) State and test the hypotheses about the Suncreen by Person Interaction Effects. Use $\alpha = 0.05$.

(iii) Estimate the variance of the effects of the random factor.

[6+5+2]

B6. Siyaso Co. wished to set up a control chart to monitor the proportion p of defective items that are produced by one of its machines. Twenty eight independent samples, each of size 50 items, were taken when the production process was in statistical control. The 28 samples were found to contain a total of 301 defective items.

(a) (i) Establish a control chart, with 0.05 probability limits, to monitor future production.

(ii) What is the average run length of the control chart when the process is in statistical control? Explain the figure to a layman.

(iii) Comment on the state of the production process when a new sample of size 50 contains 24 defective cans.

[6+3+3]

(b) Estimate the average run length of the control chart when the process shifts to $p = 0.3$. Explain the estimate to a layman. [5+3]

B7. (a) Twenty five independent samples, each of size 5, were taken from a certain production process when it was in statistical control. The objective was to establish $\bar{X}$ and R control charts for the process. The analysis of the samples obtained the following results:

$$\sum_{i=1}^{25} \bar{X}_i = 1850.028 \qquad \sum_{i=1}^{25} R_i = 0.581$$

(i) Set up an $\bar{X}$ control chart using these data. Comment on the state of the process if for the 26^{th} sample $\bar{X} = 74.023$.

(ii) The specification limits on the characteristic X are 74.000 ± 0.05. Assuming that X is a normally distributed random variable, calculate the process capability. Explain the figure to a layman.

[6+8]

(b) Consider a four-component system that functions when both components 1, 4, and at least one of the other components function. Let p_i represent component i functions. What is the reliability of the system? [6]

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APPENDIX: Constants for Constructing Quality Control Charts

Sample size	$\bar{X}$ Control Chart	R Control Chart	
n	A_2	D_3	D_4
2	1.8806	0.000	3.267
3	1.0231	0.000	2.575
4	0.7285	0.000	2.282
5	0.5768	0.000	2.115
6	0.4833	0.000	2.004
7	0.4193	0.076	1.924
8	0.3725	0.136	1.864
9	0.3367	0.184	1.816
10	0.3082	0.223	1.777

END OF QUESTION PAPER