NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 5252: Industrial Statistics

May–June 2004 **Time: 3hours**

Answer any **four** questions. All questions carry **equal** marks

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1 An engineer is designing a battery for use in a device that will be subjected to some extreme variations in temperature. The only design parameter that he can select at this point is the plate material for the battery, and he has three possible choices. When the device is manufactured and is shipped to the field, the engineer has no control over the temperature extremes that the device will encounter, and he knows from experience that temperature will probably impact the effective life of the battery. The engineer decide to test all tree materials at three temperature levels 15°C, 70°C and 125°C as these temperature levels are consistent with the product end-use environment. Four batteries are tested at each combination of plate material and temperature, and all the 36 tests are run in random order. The battery life in hours after the experiment was conducted is shown below.

Material Type	Temperature (°C)					
	15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

(a) From the above experiment identify and give reasons the:

(i) Experimental units;

(ii) The response variable;

(iii) A suitable design for the experiment. **[5 marks]**

(b) Check for the presence of temperature-material type interaction by a suitable sketch graph.

[5 marks]

(c) Analyse the data using the Analysis of Variance (ANOVA) and draw conclusions, using $\propto = 0.05$.

[15 marks]

2 (a) As part of a self-charting program at his company Mr Phil wants to construct $\overline{X}$ and R charts at the filling operation for the 16g cornflakes product. Engineers have studied the filling operation and have determined that when operating properly, boxes average 16.1g and hourly samples of 20 boxes each have sample range that average 2.22g. The data from 12 hourly samples that Mr Phil has taken are as below.

Sample Number	Sample mean (g)	Sample range (g)
1	16.2	2.0
2	15.9	2.1
3	16.3	1.8
4	16.4	3.0
5	15.8	3.5
6	15.9	3.1
7	16.0	2.9
8	16.1	1.8
9	16.3	1.5
10	16.3	1.0
11	16.4	1.0
12	16.5	0.9

(i) Compute the upper and lower limits for the $\overline{X}$ and R charts.
(ii) Plot the sample means and ranges on the $\overline{X}$ and R control charts.
(iii) What can Mr Phil conclude about the filling operation?

[5+5+5 marks]

(b) Bearing Specialist Company (BSC) Ltd produces ball and roller bearings of various sizes for automobile manufactures. One such ball bearing, the "2 tonne" bearing has been subject of numerous customer complains in recent months because of surface defects. Moyo the manager of the company's quality control department has decided that an acceptance plan based on random samples should be established for this product. He carefully searched for past periods records when the "2 tonne" bearing was known to be operating properly, and he found that 2% of the bearings were defective. If a sample of size 200 bearings and a one –tailed significant level of 0.025 is to be used;

(i) Set the acceptance criterion for the percentage defectives in the above sample.

(ii) If another sample of 200 is drawn at random and has 7 defective ball bearings, should the sample be accepted?

[5+5 marks]

3 (a) What are the three basic principles of experimental design, and for each briefly explain how they are used in designing statistical experiments. State also merits and demerits of each.

[6 marks]

(b) An experiment was conducted to compare the effect of three different insecticides(**A,B,C**) on the number of seedlings, of a particular variety of string bean, that emerged per subplot. Four different plots (in terms of moisture content, fertility, etc) of the same size were prepared, with each plot divided into three subplots of the same size. A suitable distance was maintained between the subplots within a plot. Each subplot was planted with 100 seeds and then maintained under the insecticide randomly assigned to it.

(i) Identify the experimental units, the treatments and the response variable of interest in the experiment.

[3 marks]

(ii) What was the blocking factor in the experiment and why?

[1 marks]

(iii) Let P_{ij} (i = 1,2,3,4, j = 1,2,3) be the j^{th} subplot in the i^{th} plot. Show the layout of the design of the experiment. Use the following sets of random numbers to randomize your experiment: {2,1,3}, {2,3,1}, {1,2,3} and {3,2,1}

[5 marks]

(c) The data collect from the experiment is given below:

(i) Ignore the plots and run a one-way analysis of variance (ANOVA) to compare the three insecticides.

[4marks]

(ii) Run a two-way (ANOVA) with Plot as a second factor, to compare the three insecticides and comment on your results.

[6 marks]

	Plot			
Insecticide	1	2	3	4
A	56	49	65	60
B	84	78	94	93
C	80	72	83	85

4 Control charts for $\overline{X}$ and S have been maintained on a process and have exhibited statistical control. The sample size is $n = 6$. The control chart parameters are as follows;

$\overline{X}$ chart	S chart
UCL = 708.20	UCL = 3.420
Center Line = 706.00	Center Line = 1.738
LCL = 703.80	LCL = 0.052

(a) Estimate the mean and standard deviation of the process;
(b) Estimate the natural tolerance limit for the process;
(c) Assume that the process output is well modeled by a normal distribution. If specifications are 703 and 709,estimate the fraction non-conforming;
(d) Suppose the process mean shifts to 702.00 while the standard deviation remains constant. What is the probability of an out of control signal occurring on the first sample following the shift?
(e) For the shift in (d) above, what is the probability of detecting the shift by at least the third subsequent sample?

[3+5+7+5+5]

5 (a) A 2^2 factorial experiment (with two replicates) was concerned with the problem of trying to obtain a new type of coating material for metals used in making aircraft components. The purpose of coating the metal is to increase its strength. The effects of two factors were studied, with each factor at two levels. The two factors were

A: type of coating powder;
B: Size of the powder particles.
The metal strength scores were as shown below:
(1) = 1.4, a = 1.2, b = 3.6 and ab = 1.2.

Given that SSTO = 3.11, analyse the data and draw conclusions.

[10 marks]

(b) Two factors are thought to influence the taste of Coca-Cola;
-carbonation level (A)-levels A_1 and A_2;
-Temperature (B) – levels B_1 and B_2;
An experiment is to be conducted to determine the combination of the levels of A and B which obtains the best tasting Coca-Cola. Eight random samples of raw materials (S_1, S_2, ...S_8) are available for experimentation. A test sample of Coca-Cola will be made using a sample of raw materials at a combination. The test sample will be given to a test panel of 20 people. Each tester will assign point-taste score from 1 to 10 to the test sample. The combination that obtains the best tasting Coca-Cola will be the one that gets the highest total score.

(i) If the 8 samples of raw material are from the same batch, which design is appropriate for the experiment and why? Randomize the experiment using any of the sets of random numbers below. Indicate the set(s) used.

(ii) If the samples S_1, S_2, S_3, S_4 are from a batch sourced from supplier X and that the remainder are from a batch sourced from supplier Y, which design is appropriate for the experiment and why? Randomize the experiment using any of the sets of random numbers below. Indicate the set(s) used.

(iii) Set up skeleton ANOVA tables (which display the sources of variation and their corresponding degrees of freedom) for your designs in (i) and (ii).

Set 1: {10,1,11,16,2,9,14,7,5,13,8,15,6,12,4,3}
Set 2: {2,16,7,10,3,8,9,14,12,11,4,5,6,13,1,15}
Set 3: {13,14,6,15,11,1,7,8,5,2,4,9,12,10,3,16}
Set 4: {11,2,4,16,1,14,10,7,6,5,13,15,12,9,3,8}
Set 5: {3,4,12,6,15,8,13,5,7,14,9,2,16,11,1,10}

[4 + 4 + 7 marks]

6 Frozen orange juice is packed in 250ml cans. These cans are formed on a machine by spinnnig them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether when filled it could possibly leak on the side seam or around the bottom joint. Such a can is defective. We wish to set up a control chart to improve the proportion of defective cans produced by this machine. Twenty eight samples, each of size 50, were taken when the process was in control. The 28 samples were found to contain a total of 301 defective cans.

(a) Establish a control chart, with 3-*sigma* control limits, to monitor future production. What is the corresponding *np* control chart? Comment on the state of the production process when a new sample of size 50 contains 24 defective cans.

(b) Estimate the average run length of the control chart when the process is out of control, that is when the process shifts to $p = 0.3$. Explain the estimate to a layman.

[15 + 10marks]

END OF QUESTION PAPER