NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 5252/5152: Industrial Statistics

December 2005 Exams

Time 3 hours

Answer any **four** questions. All questions carry **equal** marks

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1 Steel is normalized by heating it above the critical temperature, soaking, and then cooling. The process increases the strength of the steel. An experiment is to be conducted to determine the effect of the temperature and the heat treatment time on the strength of the normalized steel. Two temperatures (A1, A2) and two heat treatment times (B_1, B_2) will be investigated. The experiment will be performed by heating the oven to a randomly selected temperature (either A_1 and A_2) and inserting two specimens. One of the specimens will be removed after time B_1 and the other after time B_2. Then the temperature will be changed to the other level and the whole process repeated with another set of two specimens.

(a) Identify the experimental unit, the treatments and the response in the proposed experiment.

(b) The experiment is to be replicated two times. Two laboratories (L_1, L_2) each of which can run the experiment once are available for the conduct of the experiment. Suggest an appropriate design for the experiment. **Give reasons for your answer!!**

(c) The labels for the L_1 specimens are S_{11}, S_{12}, S_{13}, S_{14} and those for the L_2 specimens are S_{21}, S_{22}, S_{23}, S_{24}. Identify your treatments by the characters: T_1, T_2....., and hence randomize the experiment you proposed in part (b) using the sets of random numbers given below. **Indicate the set(s) used and how you randomize the experiment!!**

Set 1: {10,1,11,16,2,9,14,7,5,13,8,15,6,12,4,3}
Set 2: {2,16,7,10,3,8,9,14,12,11,4,5,6,13,1,15}
Set 3: {13,14,6,15,11,1,7,8,5,2,4,9,12,10,3,16}
Set 4: {11,2,4,16,1,14,10,7,6,5,13,15,12,9,3,8}

(d) Set up a skeleton ANOVA table which displays the source of variation and their corresponding degrees of freedom for the experiment in part (b).

[4 + 6 + 6 + 4marks]

2 (a) A food-processing company has tested 6 different formulations of a new soft drink (Musimboti) obtained from combining one of the 3 levels of sweetness and one of two levels of energy content. Each of six randomly chosen individuals rated (% rating) the 6 formulations when bottled in brown bottles and also when bottled in green bottles. The row data and some partial MINITAB output is as bellow. Assume that the color of the bottle is a nuisance factor, and that the only interaction that may be present are the sweetness by energy interactions. Analyze the data completely and report your findings to the company. Use $\alpha = 0.05$

		Bottle			
		Brown Energy		Green Energy	
		Low	High	Low	High
Sweetness	1	59.5	42.5	54.5	40.1
	2	66.8	49.6	64.7	50.1
	3	52.0	39.3	35.1	30.2

Analysis of variance for rating

Source	**DF**	**SS**	**MS**	**F**
Bcolour	...	102.08	....	...
Sweetness	...	696.86	.....	....
Energy	...	544.05	....	....
Sweetness*Energy	....	.....	.....	.....
Error	...	99.84	....	....
Total	...	1475.52		

[10 marks]

(b) An experiment was conducted to compare the fat content (in grams per 250 ml container) of four brands of yogurt. A randomized block design was used for the experiment with the four testing laboratories as the blocks. It was suspected that there might be a "Yogurt by Laboratory" interaction and hence each laboratory tested two containers of each brand of yogurt. The results follow:

	Lab A	Lab B	Lab C	Lab D
Yogurt 1	3.7, 4.6	2.8, 4.7	3.1, 5.0	3.3, 4.4
Yogurt 2	4.0, 5.1	2.5, 4.7	4.7, 5.9	3.0, 4.0
Yogurt 3	5.4, 3.5	4.4, 3.4	4.0, 2.9	3.2, 3.3
Yogurt 4	4.9, 5.8	8.6, 7.5	5.0, 3.3	5.8, 8.9

(i) Suppose that the fixed effects model is appropriate for the experiment. Perform an ANOVA on the data and draw conclusions. Use the 0.05 level of significance.

(ii) Based on your conclusions in (i), is it appropriate to perform pairwise comparisons of the true mean fat content of the brands of yorghut? Explain. If so, what would be the list significant difference (LSD) for the pairwise comparisons?

[6 +4marks]

3 Suppose that $X_1, X_2, \ldots X_n$ represent the quality characteristic measurements on a random sample of n units of a product of a production process. Let $\theta(X_1, X_2, \ldots X_n)$ be a sample statistic that measures the quality characteristic of the product, and suppose that when the production process is in statistical control, the mean and the variance of $\theta(X_1, X_2, \ldots X_n)$ is μ_θ and σ_θ^2.

(a) Describe how a control chart with $\alpha \in (0,1)$ probability limits is constructed and used.

(b) Suppose that the common distribution of the X_is is normal with mean 230 and the standard deviation of 20, and that n = 16 and $\theta(X_1, X_2 \ldots X_n)$ is the sample average. If $\alpha = 0.05$, what can you conclude about the production process if the average of a random sample of 16 units is 250.

[10 + 10marks]

4 Frozen orange juice is packed in 250ml cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether when filled it could possibly leak on the side seam or around the bottom joint. Such a can is defective. We wish to set up a control chart to reduce the proportion of defective cans produced by this machine. Twenty eight samples, each of size 50, were taken when the process was in control. The 28 samples were found to contain a total of 301 defective cans.

(a) Establish a control chart, with 3-*sigma* control limits, to monitor future production. What is the corresponding *np* control chart? Comment on the state of the production process when a new sample of size 50 contains 24 defective cans.

(b) Estimate the average run length of the control chart when the process is out of control, that is when the process shifts to $p = 0.3$. Explain the estimate to a layman.

[14 + 6marks]

5 A company that dyes rugs is interested in establishing a control chart for the mean color when dyeing solid-colored rugs. Although maintaining uniform color is important for patterned or multi-colored rugs, it is much more important for solid –colored rugs, where minor changes in solid colors are easily recognizable. Rug color quality can be monitored by taking readings on a calorimeter. Twenty five samples of five measurements each from a rug being dyed yielded the data listed below. These data were obtained while the manager believed that the process was in statistical control.

Sample No.	Sample Sum	Sample Range	Sample No.	Sample Sum	Sample Range
1	8.4	1.8	16	13.5	1.4
2	12.1	1.9	17	10.3	3.2
3	7.6	2.1	18	10.5	1.2
4	9.6	2.5	19	10.2	2.8
5	11.5	1.6	20	12.5	2.8
6	13.1	1.6	21	7.5	1.5
7	12.9	2.6	22	8.8	1.3
8	11.2	1.9	23	8.6	1.8
9	13.1	1.2	24	6.4	2.0
10	10.8	3.4	25	12.3	4.0
11	10.0	3.4			
12	9.5	0.7			
13	8.7	3.8			
14	8.7	2.2			
15	10.4	3.5			

(a) Set up the $\overline{\overline{X}}$ control chart, with the 0.01 probability limits for the process. Comment on the state of the process if for the 26^{th} sample of size 5 has $\overline{X} = 3$.

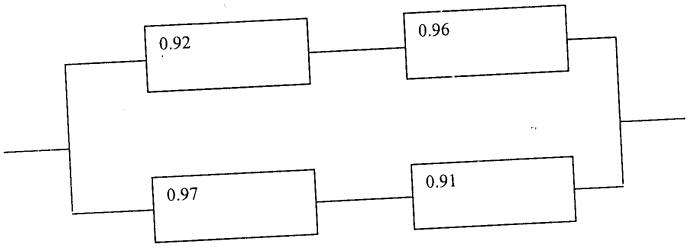
(b) Suppose that rug quality is a random variable which has a normal distribution with mean 2.07 and standard deviation 0.98. If the specification limits on the color are 2 ± 0.5, calculate the process capability. Explain the figure to a layman

[10+10marks]

6 (a) The diagram below shows a mixed system. In the system the components function independently of each other with reliabilities given in the boxes.
Calculate the reliability of the system.

(b) Consider a 2-Out of 3 system with independent components. If the reliability of each component is p, what is the reliability of the system?

(c) Consider a four component system that functions when both components 1, 4 functions and at least one of the other components function. Let pi represent functioning of component i . What is the reliability of the system?



[2+3+5 marks]

(d) Suppose that a series system has two components, and that the components function independently of each other for an amount of time (in hours) uniformly distributed over the interval (0,100). Find the system's

(i) reliability function,
(ii) failure rate function and
(iii) expected life.

[4+3+3 marks]

END OF QUESTION PAPER