

**NATIONAL UNIVERSITY OF SCIENCE AND
TECHNOLOGY**

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

May 2001
3 Hours

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This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.

Throughout this paper a_t represents white noise, $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A : Answer ALL questions from this section.

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(0,1,2) process.

[3 Marks]

2. A series of data follows an ARIMA(1,1,1) model, with $\phi_1 = 0.4$, $\theta_1 = -0.3$ and $C = 4.0$. Calculate forecasts for $t=101$ and $t=102$, given that $z_{99} = 79$, $z_{100} = 88$ and $f_{100} = 85$.

[4 Marks]

3. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{l=1}^p \phi_l \rho_{k-l}, k = 1, 2, \dots$$

Hence find the first 3 terms in the autocorrelation function for an AR(2) process with $\phi_1 = 0.0$, and $\phi_2 = 0.3$.

[7 Marks]

4. Determine if the following process is stationary and/or invertible.

$$z_t - z_{t-1} = 0.2z_{t-2} + a_t - 0.4f_{t-1}.$$

[3 Marks]

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5. Comment briefly on the relative merits of time series methods and regression methods in producing sales forecasts.

[4 Marks]

6. Given (x_t, y_t) , $t = 1, 2, \dots, n$, describe how you would use a least squares method to find the best linear unbiased estimates of α and β for each of the following models,

(a) $y_t = \alpha + \beta x_t + u_t$, where $u_t = x_t^2 a_t$,

(b) $y_t = \alpha + \beta x_t + u_t$, where $u_t = 0.5u_{t-1} + a_t$.

[6 Marks]

7. In standard notation, a multiple regression model may be written as

$$y = X\beta + a.$$

(a) Show that the sum of the squares of the residuals is minimised when

$$\beta = V^{-1}X'y.$$

[3 Marks]

(b) Find and comment on the value of the determinant of the matrix V for the following information.

y_1	x_1	x_2
3	1	-1
2	2	-3
2	-1	3

[4 Marks]

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SECTION B : Answer TWO questions from this section.
Each question carries 33 marks.

(a) An ARIMA(0,0,2) process

$$z_t - \mu = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

has been fitted by an ARIMA(1,0,0) model such that $E(u_t^2)$ has been minimised, where

$$u_t = z_t - \mu - \phi_1(z_{t-1} - \mu).$$

- i. Show that u_t follows an ARIMA(0,0,3) process. [3 Marks]
- ii. Given that $\theta_1 = 0.5$ and that $\theta_2 = -0.5$, derive the autocorrelation function of u_t . [7 Marks]
- iii. The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^k \phi_{k\ell} \rho_{j-\ell} \quad j = 1, 2, \dots, k.$$

Use these equations to find the partial autocorrelation function, ϕ_{kk} , of u_t up to lag 3. [6 Marks]

- iv. Find the coefficients up to lag 3 in the $AR(\infty)$ representation of u_t . [5 Marks]

(b) Derive the autocorrelation function for an ARIMA(0,0,1) \times (0,0,1) $_4$ process. [5 Marks]

- (c) i. Write an ARIMA(1,0,1) process in general linear form. [3 Marks]
- ii. Given that, if a stationary model is written in general linear process form, then the covariance $E((z_t - \mu)(z_{t-k} - \mu))$ can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell} \psi_{\ell} \sigma^2,$$

find the autocorrelation function for an ARIMA(1,0,1) process. [4 Marks]

8. An operations research consultant has been asked to analyse the mean monthly humidity levels for a city situated close to the equator. The consultant used the MINITAB statistical package to produce the output given in appendix A.

For each MINITAB command given, briefly explain

- (a) the reasons for using the command at that stage in the analysis,
- (b) the conclusions reached from the output to that command.

[28 Marks]

Use the output to estimate humidity levels for the next two months.

[5 Marks]

9. A statistical consultant has been asked to analyse monthly data on the sales of Mealie Meal, taking into account the price indices of Mealie Meal, Rice and Meat, as well as the income index, Y . The consultant used the MINITAB statistical package to produce the output given in appendix B.

For each MINITAB command given, briefly explain

- (a) the reasons for using the command at that stage in the analysis,
- (b) the conclusions reached from the output to that command.

[28 Marks]

Given that the price indices of Meal, Rice and Meat are estimated to be 140, 156 and 145, respectively, and that the income index will be 90, use the output to estimate Mealie Meal sales for the next month.

[5 Marks]

END OF EXAMINATION PAPER

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