NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

May 2002
3 Hours

This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.

Throughout this paper z_t represents an observation at time t, f_t represents a forecast at time t and a_t represents white noise, $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A : Answer ALL questions from this section.

1. A series of data follows an ARIMA(1,1,1), no constant, model, with $\phi_1 = -0.2$ and $\theta_1 = -0.4$. Calculate forecasts for t=101 and t=102, given that $z_{99} = 76$, $z_{100} = 77$ and $f_{100} = 82$.

   [4 Marks]

2. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{\ell=1}^{p} \phi_\ell \rho_{k-\ell}, k = 1, 2, \ldots.$$

   Hence find the first 3 terms in the autocorrelation function for an AR(2) process with $\phi_1 = 0.3$ and $\phi_2 = 0.2$.

   [7 Marks]

3. Determine if the following process is stationary and/or invertible.

$$z_t - z_{t-1} = 0.1z_{t-2} + a_t - 0.4f_{t-1}.$$

   [3 Marks]

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4. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(0,1,2) process.

[4 Marks]

5. Comment briefly on the relative merits of time series methods and regression methods in producing sales forecasts.

[4 Marks]

6. Given (x_t, y_t), $t = 1, 2, \ldots, n$, describe how you would use a least squares method to find the best linear unbiased estimates of α and β for each of the following models,

(a) $y_t = \alpha + \beta x_t + u_t$, where $u_t = x_t a_t$,
(b) $y_t = \alpha + \beta x_t + u_t$, where $u_t = 0.7u_{t-1} + a_t$.

[6 Marks]

7. An ARIMA(0,0,1) process has been fitted by an ARIMA(1,0,0) model such that $E(u_t^2)$ has been minimised, where

$$u_t = z_t - \mu - \phi_1(z_{t-1} - \mu).$$

(a) By finding ϕ_1 in terms of the parameters of the process, show that

$$u_t = a_t - \frac{{\theta_1}^3}{1+\theta_1^2}a_{t-1} - \frac{\theta_1^2}{1+\theta_1^2}a_{t-2}.$$

[4 Marks]

(b) Derive the autocorrelation function of u_t.

[5 Marks]

SECTION B : Answer TWO questions from this section.
Each question carries 33 marks.

8. (a) Derive the autocorrelation function for an ARIMA$(0,0,1)$x$(0,0,1)_4$ process.

[5 Marks]

(b) The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^{k} \phi_{k\ell}\rho_{j-\ell} \qquad j = 1, 2, \ldots, k.$$

Use these equations to find the partial autocorrelation function, ϕ_{kk}, for an ARIMA$(0,0,1)$x$(0,0,1)_4$ process, up to lag 3.

[5 Marks]

(c) Find the coefficients up to lag 6 in the $AR(\infty)$ representation of an ARIMA$(0,0,1)$x$(0,0,1)_4$ process.

[6 Marks]

(d) Describe the autocorrelation and partial autocorrelation functions you would expect from a time series of 900 observations and its differences from an ARIMA$(0,0,1)$x$(0,1,1)_4$, noconstant, process with $\theta_1 = -0.5$ and $\Theta_1 = 0.1$.

[4 Marks]

(e) Show that an ARIMA$(0,0,1)$x$(1,0,0)_4$ model written in general linear process form is

$$z_t - \mu = \sum_{j=0}^{\infty} \psi_j a_{t-j},$$

where

$$\psi_{4p} = \Phi_1^p, \quad p = 0,1,2,\ldots,$$

$$\psi_{4p+1} = -\theta_1 \Phi_1^p, \quad p = 0,1,2,\ldots$$

and

$$\psi_j = 0, \quad \text{otherwise} \; .$$

[6 Marks]

(f) Given that, if a stationary model is written in general linear process form, then the covariance $E((z_t - \mu)(z_{t-k} - \mu))$ can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell} \psi_\ell \sigma^2,$$

find the autocorrelation function for an ARIMA$(0,0,1)$x$(1,0,0)_4$ process.

[7 Marks]

9. An operations research consultant has been asked to analyse the mean monthly rainfall for a particular country. The consultant used the MINITAB statistical package to produce the output given in appendix A.

For each MINITAB command given, briefly explain

(a) the reasons for using the command at that stage in the analysis,

(b) the conclusions reached from the output to that command.

Use the output to estimate the rainfall over the next three months.

[33 Marks]

10. A statistical consultant has been asked to analyse monthly data on the sales of Mealie Meal, taking into account the price indices of Mealie Meal, Potatoes and Chicken, as well as the income index, Y. The consultant used the MINITAB statistical package to produce the output given in appendix B.

For each MINITAB command given, briefly explain

(a) the reasons for using the command at that stage in the analysis,

(b) the conclusions reached from the output to that command.

Given that the price indices of Meal, Potatoes and Chicken are estimated to be 137, 136 and 140, respectively, for the next two months and that the income index will be 126 for the next two months, use the output to estimate Mealie Meal sales for the next two months.

[33 Marks]

END OF EXAMINATION PAPER
