NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

May 2003
3 Hours

This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.

Throughout this paper a_t represents white noise, $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A : Answer ALL questions from this section.

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(0,2,1) process and its differences.

[3 Marks]

2. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{l=1}^{p} \phi_\ell \rho_{k-\ell}.$$

Hence find the first 4 terms in the autocorrelation function for an AR(2) process with $\phi_1 = 0.2$ and $\phi_2 = -0.3$.

[7 Marks]

3. (a) Derive the autocorrelation function for an ARIMA(0,0,1)x(0,0,1)$_{12}$ process.

[5 Marks]

(b) An ARIMA(0,0,0)x(0,0,1)$_{12}$ model is fitted to this process such that the sum of the squares of the residuals is minimised. Find the autocorrelation function for these residuals.

[5 Marks]

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4. Determine if the following process is stationary and/or invertible.

$$z_t - z_{t-1} = -0.4z_{t-1} + 0.1z_{t-2} + a_t - 0.8f_{t-1} - 0.3a_{t-1}$$

[3 Marks]

5. Describe how you would use an ordinary least squares method to obtain the best linear unbiased estimates of α and β for each of the following models,

   (a) $y_t = \alpha + \beta x_t + u_t$, where $u_t = {x_t}^2 a_t$,

   (b) $y_t = \alpha {x_t}^\beta \exp u_t$, where $u_t = x_t a_t$.

[6 Marks]

6. Comment on the effect on the estimation of parameters in a regression model of autocorrelated residuals of lag one. Describe a technique for dealing with this violation of the assumptions of the ordinary least squares model.

[5 Marks]

SECTION B : Answer TWO questions from this section.
Each question carries 33 marks.

7. (a) Write an ARIMA(1,0,2) model in general linear process form,

$$z_t - \mu = \sum_{j=0}^{\infty} \psi_j a_{t-j}.$$

[5 Marks]

   (b) Show that, if a stationary model is written in general linear process form, then the covariance $E((z_t - \mu)(z_{t-k} - \mu))$ can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell}\psi_\ell \sigma^2.$$

[4 Marks]

(c) Hence show that the autocorrelation function for an ARIMA(1,0,2) process is

$$\rho_k = \begin{cases} \frac{B(1-\phi_1^2)(1+A)+\phi_1 A^2}{(1+B^2)(1-\phi_1^2)+A^2} & ; \quad k = 1 \\ \frac{A(1-\phi_1^2)(\phi_1^{k-2}+B\phi_1^{k-1})+\phi_1^k A^2}{(1+B^2)(1-\phi_1^2)+A^2} & ; \quad k > 1 \end{cases}$$

where $B = \phi_1 - \theta_1$ and $A = \phi_1 B - \theta_2$.

[8 Marks]

(d) The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^{k} \phi_{k\ell}\rho_{j-\ell} \qquad j = 1, 2, \ldots, k.$$

Use these equations to find the partial autocorrelation function, ϕ_{kk}, for an ARIMA(1,0,2) process, up to lag 3, given that $\theta_1 = 0, \theta_2 = 0.2$ and $\phi_1 = 0.3$.

[8 Marks]

(e) Describe the autocorrelation and partial autocorrelation functions you would expect from a time series of 400 observations and its differences from an ARIMA(1,2,2) , noconstant, process with $\theta_1 = 0, \theta_2 = 0.2$ and $\phi_1 = 0.3$.

[3 Marks]

(f) Given the following values from a time series following an ARIMA(1,2,2) , noconstant, model with $\theta_1 = 0, \theta_2 = 0.2$ and $\phi_1 = 0.3$, calculate forecasts for the next 2 periods.

t	z_t	f_t
398	17.84	17.67
399	17.73	16.87
400	18.39	17.92

[5 Marks]

8. An operations research consultant has been asked to analyse daily data on the traffic intensity levels along a stretch of new road in a city. The purpose is to produce a model so that forecasts can be produced for traffic intensity. The consultant used the MINITAB statistical package to produce the output given in appendix A.

   (a) Explain clearly the output given, including

      i. the reasons for using each of the commands,

      ii. a summary of the conclusions of the output from each command.

 [25 Marks]

(b) Using the final model used by the consultant, calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the city council.

[8 Marks]

9. A statistical consultant has been asked to analyse daily data on electricity consumption in a particular city in the North Africa. The purpose is to produce a model so that forecasts can be produced for electricity consumption on a particular day with an estimated temperature. The consultant used the MINITAB statistical package to produce the output given in appendix B.

(a) Explain clearly the output given, including

i. the reasons for using each of the commands,

ii. a summary of the conclusions of the output from each command.

[25 Marks]

(b) On the basis of the information given, specify a model which you think is most appropriate for these data. Use this model to calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the electricity authority.

[8 Marks]

END OF EXAMINATION PAPER
