

NATIONAL UNIVERSITY OF SCIENCE AND  
TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

July 2003 Supplementary Examination  
3 Hours

This paper contains TWO sections. Answer ALL the questions in section A  
and TWO questions from section B.

Throughout this paper  $a_t$  represents white noise,  $E(a_t) = 0$  and  $E(a_t^2) = \sigma^2$ .

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SECTION A : Answer ALL questions from this section.

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(0,1,2) process and its differences.

[3 Marks]

2. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{\ell=1}^p \phi_{\ell} \rho_{k-\ell}$$

Hence find the first 4 terms in the autocorrelation function for an AR(1) process with  $\phi_1 = 0.2$ .

[7 Marks]

3. (a) Derive the autocorrelation function for an ARIMA(0,0,1)x(0,0,1)<sub>12</sub> process.

[5 Marks]

- (b) An ARIMA(0,0,0)x(0,0,1)<sub>12</sub> model is fitted to this process such that the sum of the squares of the residuals is minimised. Find the autocorrelation function for these residuals.

[4 Marks]

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4. Determine if the following process is stationary and/or invertible.

$$z_t - 2z_{t-1} = -0.4z_{t-1} + 0.1z_{t-2} + a_t - 0.6f_{t-1} - 0.2a_{t-1}$$

[3 Marks]

5. Describe how you would use an ordinary least squares method to obtain the best linear unbiased estimates of  $\alpha$  and  $\beta$  for each of the following models,

(a)  $y_t = \alpha + \beta x_t + u_t$ , where  $u_t = x_t a_t$ ,

(b)  $y_t = \alpha x_t^\beta \exp u_t$ , where  $u_t = x_t^2 a_t$ .

[7 Marks]

6. Comment on the effect on the estimation of parameters in a regression model of autocorrelated residuals of lag one. Describe a technique for dealing with this violation of the assumptions of the ordinary least squares model.

[5 Marks]

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**SECTION B : Answer TWO questions from this section.**  
Each question carries 33 marks.

7. (a) Write an ARIMA(1,0,1) model in general linear process form,

$$z_t - \mu = \sum_{j=0}^{\infty} \psi_j a_{t-j}.$$

[6 Marks]

- (b) Show that, if a stationary model is written in general linear process form, then the covariance  $E((z_t - \mu)(z_{t-k} - \mu))$  can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell} \psi_{\ell} \sigma^2.$$

[4 Marks]

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(c) Hence find the autocorrelation function for an ARIMA(1,0,1) process.

[6 Marks]

(d) The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^k \phi_{k\ell} \rho_{j-\ell} \quad j = 1, 2, \dots, k.$$

Use these equations to find the partial autocorrelation function,  $\phi_{kk}$ , for an ARIMA(1,0,1) process, up to lag 3, given that  $\theta_1 = 0$ , and  $\phi_1 = 0.3$ .

[9 Marks]

(e) Describe the autocorrelation and partial autocorrelation functions you would expect from a time series of 400 observations and its differences from an ARIMA(1,2,1), nonconstant, process with  $\theta_1 = 0$  and  $\phi_1 = 0.3$ .

[3 Marks]

(f) Given the following values from a time series following an ARIMA(1,2,1), nonconstant, model with  $\theta_1 = 0$  and  $\phi_1 = 0.3$ , calculate forecasts for the next 2 periods.

$t$	$z_t$	$f_t$
398	17.84	17.67
399	17.73	16.87
400	18.39	17.92

[5 Marks]

8. An operations research consultant has been asked to analyse daily data on the traffic intensity levels in a particular city. The purpose is to produce a model so that forecasts can be produced for traffic intensity. The consultant used the MINITAB statistical package to produce the output given in appendix A.

- (a) Explain clearly the output given, including
- the reasons for using each of the commands,
  - a summary of the conclusions of the output from each command.

[25 Marks]

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- (b) Using the final model used by the consultant, calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the city council.

[8 Marks]

9. A statistical consultant has been asked to analyse daily data on electricity consumption in a particular city in the North Africa. The purpose is to produce a model so that forecasts can be produced for electricity consumption on a particular day with an estimated temperature. The consultant used the MINITAB statistical package to produce the output given in appendix B.

- (a) Explain clearly the output given, including
- i. the reasons for using each of the commands,
  - ii. a summary of the conclusions of the output from each command.

[25 Marks]

- (b) On the basis of the information given, specify a model which you think is most appropriate for these data. Use this model to calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the electricity authority.

[8 Marks]

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END OF EXAMINATION PAPER

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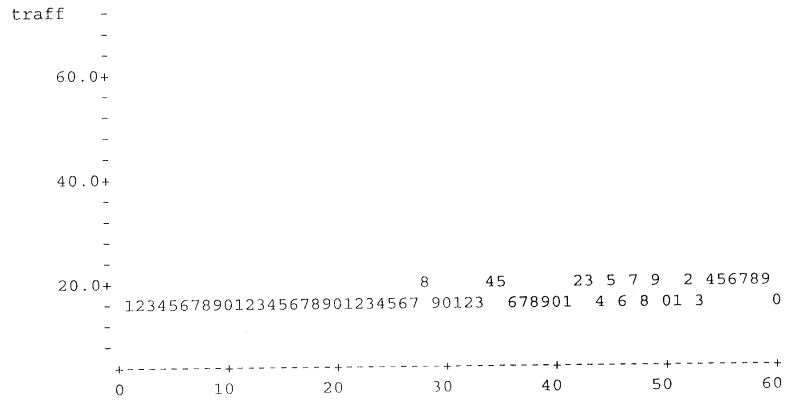
Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$H(t - c)$
$e^{-cs}F(s)$	$f(t - c)H(t - c)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s + a)$	$e^{-at}$
$1/(s + a)^{n+1}$	$t^n e^{-at}/n!$
$\frac{k}{s^2 + k^2}$	$\sin kt$
$\frac{s}{s^2 + k^2}$	$\cos kt$
$\frac{k}{s^2 - k^2}$	$\sinh kt$
$\frac{s}{s^2 - k^2}$	$\cosh kt$
$\frac{1}{(s^2 + k^2)^2}$	$\frac{1}{2k^3}[\sin kt - kt \cos kt]$
$\frac{s}{(s^2 + k^2)^2}$	$\frac{1}{2k} t \sin kt$
$\ln(1 + 1/s)$	$(1 - e^{-t})/t$
$\ln[(s + k)/(s - k)]$	$2(\sinh kt)/t$
$\ln(1 - k^2/s^2)$	$2(1 - \cosh kt)/t$
$\ln(1 + k^2/s^2)$	$2(1 - \cos kt)/t$
$\tan^{-1}(k/s)$	$\sin kt/t$

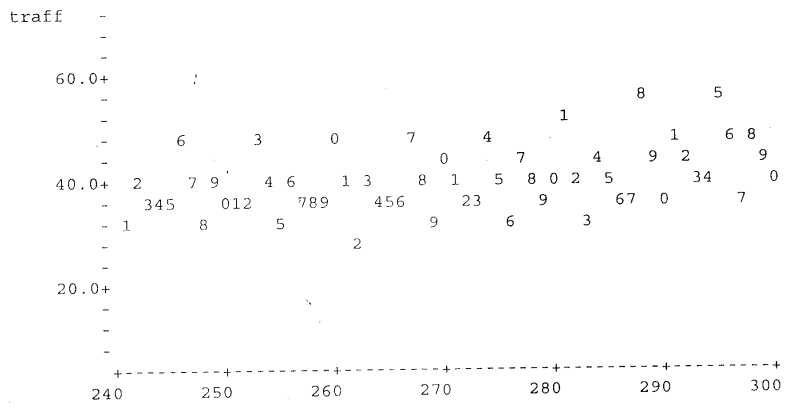
Appendix A

MTB> name c1 'traff'

MTB > tsplot c1



...





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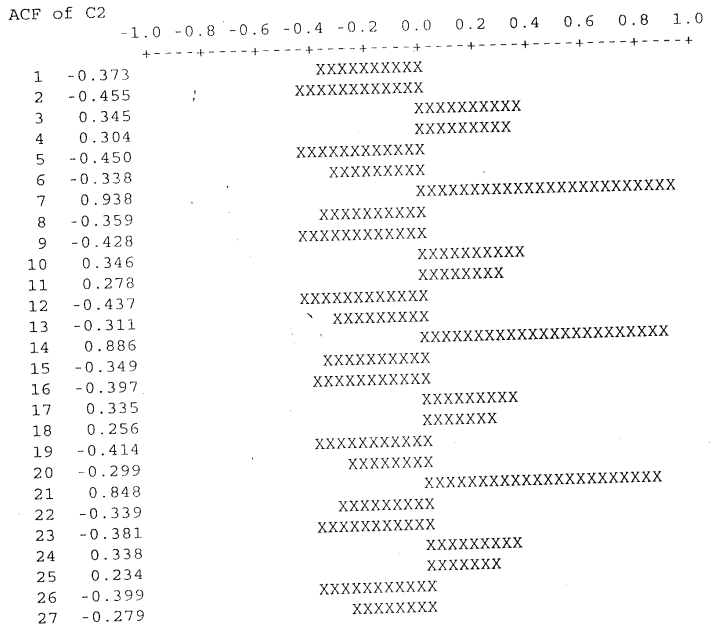
7 0.948 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
8 0.798 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
9 0.758 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
10 0.847 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
11 0.835 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
12 0.733 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
13 0.765 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
14 0.897 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
15 0.756 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
16 0.721 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
17 0.807 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
18 0.792 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
19 0.695 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
20 0.725 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
21 0.851 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
22 0.715 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
23 0.683 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
24 0.766 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
25 0.748 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
26 0.654 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
27 0.683 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
28 0.803 XXXXXXXXXXXXXXXXXXXXXXXXXXXX
29 0.672 XXXXXXXXXXXXXXXXXXXXXXXXXXXX

```

```

MTB > diff c1 c2
MTB > acf c2

```





```

28 0.808
29 -0.326
XXXXXXXXXXXXXXXXXXXXXXXXXXXX
XXXXXXXXXXXX

```

```

MTB > diff 7 c1 c3
MTB > acf c3

```

ACF of C3

```

-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
+-----+-----+-----+-----+-----+-----+
1 0.326 XXXXXXXXXX
2 0.159 XXXXX
3 0.178 XXXXX
4 0.010 X
5 0.003 X
6 0.007 X
7 -0.179 XXXXX
8 -0.057 XX
9 -0.010 X
10 0.007 X
11 -0.043 XX
12 -0.075 XXX
13 0.008 X
14 -0.045 XX
15 -0.086 XXX
16 -0.087 XXX
17 -0.070 XXX
18 -0.032 XX
19 0.017 X
20 -0.007 X
21 0.071 XXX
22 0.098 XXX
23 0.168 XXXXX
24 0.175 XXXXX
25 0.118 XXXX
26 0.038 XX
27 0.007 X
28 0.011 X
29 -0.036 XX

```

```

MTB > pacf c3

```

PACF of C3

```

-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
+-----+-----+-----+-----+-----+-----+
1 0.326 XXXXXXXXXX
2 0.058 XX
3 0.124 XXXX
4 -0.100 XXX
5 0.005 X
6 -0.008 X
7 -0.189 XXXXXX
8 0.060 XXX
9 0.018 X
10 0.070 XXX
11 -0.095 XXX

```

12	-0.059	XX
13	0.068	XXX
14	-0.088	XXX
15	-0.046	XX
16	-0.065	XXX
17	0.033	XX
18	-0.005	X
19	0.012	X
20	0.009	X
21	0.074	XXX
22	0.042	XX
23	0.100	XXX
24	0.079	XXX
25	0.022	XX
26	-0.060	XXX
27	-0.047	XX
28	0.039	XX
29	-0.036	XX

MTB > arima 1 0 0 0 1 1 7 c1 c20 c30;  
SUBC> const.

Estimates at each iteration

Iteration	SSE	Parameters		
0	672.826	0.100	0.100	0.732
1	628.362	0.250	0.176	0.570
2	620.658	0.337	0.227	0.477
3	620.642	0.341	0.227	0.471
4	620.642	0.341	0.227	0.471

Relative change in each estimate less than 0.0010

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.3410	0.0479	7.12
SMA 7	0.2269	0.0509	4.46
Constant	0.47088	0.04947	9.52

Differencing: 0 regular, 1 seasonal of order 7  
No. of obs.: Original series 398, after differencing 391  
Residuals: SS = 620.473 (backforecasts excluded)  
MS = 1.599 DF = 388

Modified Box-Pierce chi-square statistic

Lag	12	24	36	48
Chi-square	19.7 (DF=10)	33.8 (DF=22)	47.6 (DF=34)	57.5 (DF=46)

MTB > let c21=c20\*c20  
MTB > regr c21 1 c30

The regression equation is  
C21 = - 1.83 + 0.104 C30

391 cases used 7 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	-1.8257	0.3876	-4.71	0.000
C30	0.10385	0.01097	9.46	0.000

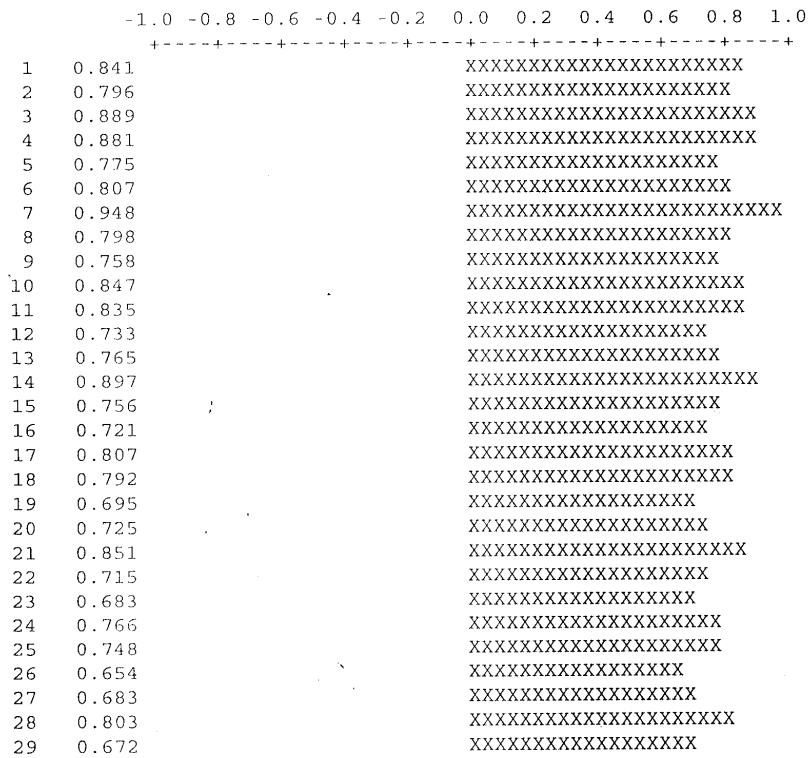
s = 2.814      R-sq = 18.7%      R-sq(adj) = 18.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	709.05	709.05	89.57	0.000
Error	389	3079.52	7.92		
Total	390	3788.57			

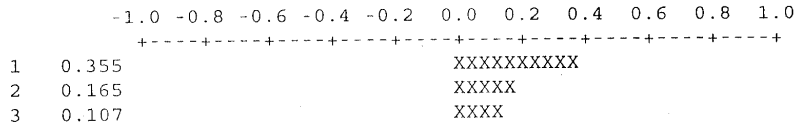
MTB > let c10=log(c1)  
MTB > acf c10

ACF of c10



MTB > diff 7 c10 c11  
MTB > acf c11

ACF of C11





```

25 -0.059
26 -0.014
27 -0.047
28 0.005
29 -0.019

```

```

XX
X
XX
X
X

```

```

MTB > arima 1 0 0 0 1 1 7 c10 c20 c30;
SUBC> const.

```

```

Estimates at each iteration
Iteration   SSE      Parameters
0           4.40891   0.100   0.100   0.110
1           1.21445   0.250   0.181   0.053
2           0.46034   0.346   0.285   0.016
3           0.45902   0.345   0.301   0.015
4           0.45900   0.343   0.295   0.015
5           0.45900   0.344   0.295   0.015

```

Relative change in each estimate less than 0.0010

```

Final Estimates of Parameters
Type      Estimate   St. Dev.  t-ratio
AR 1      0.3436      0.0477    7.21
SMA 7     0.2953      0.0490    6.02
Constant  0.014595    0.001233  11.84

```

```

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.: Original series 398, after differencing 391
Residuals:   SS = 0.458926 (backforecasts excluded)
              MS = 0.001183 DF = 388

```

```

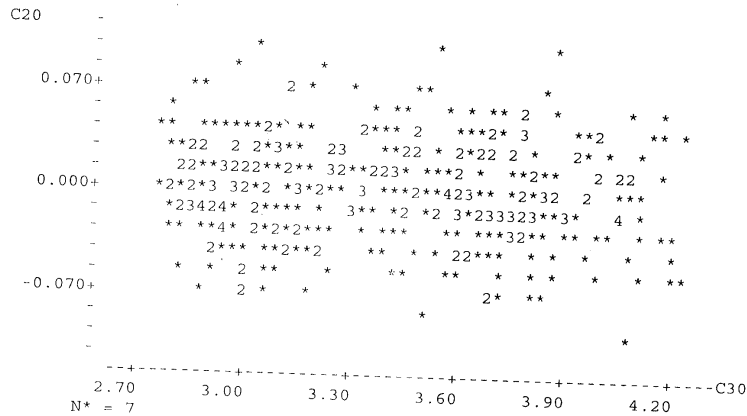
Modified Box-Pierce chisquare statistic
Lag      12      24      36      48
Chisquare 7.6 (DF=10) 21.6 (DF=22) 30.3 (DF=34) 38.6 (DF=46)

```

```

MTB > plot c20 c30

```



MTB > acf c20

ACF of C20

	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
1	-0.014					X					
2	0.018					X					
3	0.085					XXX					
4	-0.031					XX					
5	-0.016					X					
6	0.017					X					
7	-0.001					X					
8	-0.057					XX					
9	-0.025					XX					
10	0.049					XX					
11	-0.006					X					
12	-0.059					XX					
13	0.026					XX					
14	-0.005					X					
15	-0.052					XX					
16	-0.021					XX					
17	-0.006					X					
18	-0.072					XXX					
19	0.052					XX					
20	-0.010					X					
21	0.041					XX					
22	-0.005					X					
23	0.036					XX					
24	0.137					XXXX					
25	-0.025					XX					
26	-0.003					X					
27	-0.016					X					
28	-0.014					X					
29	-0.014					X					

MTB > cdf 7.6;

SUBC> chisq 10.  
7.6000 0.3322

MTB > cdf 21.6;

SUBC> chisq 22.  
21.6000 0.5160

MTB > pacf c20

PACF of C20

	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
1	-0.014					X					
2	0.017					X					
3	0.085					XXX					
4	-0.029					XX					
5	-0.021					XX					
6	0.010					X					
7	0.005					X					

```

8 -0.055 XX
9 -0.031 XX
10 0.052 XX
11 0.007 X
12 -0.061 XXX
13 0.012 X
14 0.003 X
15 -0.042 XX
16 -0.035 XX
17 -0.007 X
18 -0.057 XX
19 0.054 XX
20 -0.018 X
21 0.049 XX
22 -0.009 X
23 0.032 XX
24 0.128 XXXX
25 -0.018 X
26 -0.021 XX
27 -0.039 XX
28 0.006 X
29 -0.011 X

```

```

MTB > arima 1 0 1 0 1 1 7 c10 c20 c30;
SUBC> const.

```

Estimates at each iteration

Iteration	SSE	Parameters			
0	5.35331	0.100	0.100	0.100	0.110
1	0.77064	0.250	-0.050	0.215	0.040
2	0.61559	0.400	0.070	0.221	0.028
3	0.45706	0.475	0.143	0.293	0.012
4	0.45699	0.477	0.151	0.301	0.012
5	0.45699	0.478	0.152	0.301	0.012
6	0.45699	0.478	0.153	0.302	0.012
7	0.45699	0.478	0.153	0.302	0.012

Relative change in each estimate less than 0.0010

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.4779	0.1274	3.75
MA 1	0.1529	0.1433	1.07
SMA 7	0.3016	0.0490	6.15
Constant	0.011608	0.001028	11.29

Differencing: 0 regular, 1 seasonal of order 7

No. of obs.: Original series 398, after differencing 391

Residuals: SS = 0.456056 (backforecasts excluded)

MS = 0.001178 DF = 387

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	7.1 (DF= 9)	20.7 (DF=21)	29.4 (DF=33)	37.6 (DF=45)

```

MTB > arima 1 0 0 0 1 1 7 c10 c20 c30;
SUBC> const.

```

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.3436	0.0477	7.21
SMA 7	0.2953	0.0490	6.02
Constant	0.014595	0.001233	11.84

Differencing: 0 regular, 1 seasonal of order 7  
 No. of obs.: Original series 398, after differencing 391  
 Residuals: SS = 0.458926 (backforecasts excluded)  
 MS = 0.001183 DF = 388

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	7.6 (DF=10)	21.6 (DF=22)	30.3 (DF=34)	38.6 (DF=46)

MTB > print c10 c20 c30

ROW	C10	C20	C30
1	2.73925	*	*
2	2.77592	*	*
3	2.74592	*	*
4	2.78259	*	*
5	2.75259	*	*
6	2.75592	*	*
7	2.79259	*	*
8	2.76259	0.0040161	2.75857
9	2.85783	0.0592978	2.79853
...			
384	3.72841	-0.0683545	3.79676
385	3.81829	-0.0494174	3.86771
386	4.22086	0.0476971	4.17317
387	4.07244	-0.0505411	4.12298
388	3.77334	-0.0006607	3.77400
389	4.24110	0.0607415	4.18036
390	4.01632	0.0165476	3.99977
391	3.84007	0.0560169	3.78406
392	3.86210	-0.0237489	3.88585
393	4.18091	-0.0555130	4.23642
394	4.04642	-0.0418117	4.08823
395	3.77660	-0.0025844	3.77919
396	4.27755	0.0386729	4.23888
397	4.04619	0.0076406	4.03855
398	3.85115	0.0027657	3.84839



APPENDIX B

```

MTB > name c1 'consump' c2 'temp' c10 'holiday'
MTB > name c3 'day'
MTB > set c3
DATA> 15(1:7)
DATA> end
MTB > delete 101 102 103 104 105 c3
MTB > indicators c3 c3-c9
MTB > name c3 'monday' c9 'sunday'
MTB > regress c1 9 c2-c10

```

```

*   sunday is highly correlated with other X variables
*   sunday has been removed from the equation

```

The regression equation is  

$$\text{consump} = 48.7 + 1.32 \text{ temp} + 3.09 \text{ monday} + 4.73 \text{ C4} + 5.21 \text{ C5} + 9.02 \text{ C6} + 4.60 \text{ C7} + 0.75 \text{ C8} - 9.05 \text{ holiday}$$

99 cases used 1 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	48.730	4.794	10.17	0.000
temp	1.3229	0.1128	11.73	0.000
monday	3.093	2.479	1.25	0.216
C4	4.730	2.442	1.94	0.056
C5	5.212	2.514	2.07	0.041
C6	9.016	2.521	3.58	0.001
C7	4.599	2.520	1.83	0.071
C8	0.753	2.513	0.30	0.765
holiday	-9.050	2.925	-3.09	0.003

s = 6.560      R-sq = 63.0%      R-sq(adj) = 59.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	6604.78	825.60	19.19	0.000
Error	90	3872.88	43.03		
Total	98	10477.66			

SOURCE	DF	SEQ SS
temp	1	5278.70
monday	1	37.28
C4	1	0.25
C5	1	43.75
C6	1	595.42
C7	1	209.31
C8	1	28.22
holiday	1	411.85

Unusual Observations

Obs.	temp	consump	Fit	Stdev.Fit	Residual	St.Resid
17	45.9	128.674	114.713	1.921	13.961	2.23R
23	53.0	135.956	123.594	2.225	12.362	2.00R
30	42.3	96.705	109.401	1.746	-12.696	-2.01R

53	39.8	126.639	110.352	1.769	16.287	2.58R
87	32.0	108.911	96.326	1.920	12.585	2.01R
99	46.8	121.666	104.712	3.057	16.955	2.92R

R denotes an obs. with a large st. resid.

MTB > regress c1 8 c2 c4-c10 c20 c30

The regression equation is

$$\text{consump} = 51.8 + 1.32 \text{ temp} + 1.64 \text{ C4} + 2.12 \text{ C5} + 5.92 \text{ C6} + 1.51 \text{ C7} - 2.34 \text{ C8} - 3.09 \text{ sunday} - 9.05 \text{ holiday}$$

99 cases used 1 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	51.823	4.779	10.84	0.000
temp	1.3229	0.1128	11.73	0.000
C4	1.637	2.443	0.67	0.504
C5	2.119	2.514	0.84	0.401
C6	5.923	2.520	2.35	0.021
C7	1.507	2.519	0.60	0.551
C8	-2.340	2.513	-0.93	0.354
sunday	-3.093	2.479	-1.25	0.216
holiday	-9.050	2.925	-3.09	0.003

s = 6.560      R-sq = 63.0%      R-sq(adj) = 59.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	6604.78	825.60	19.19	0.000
Error	90	3872.88	43.03		
Total	98	10477.66			

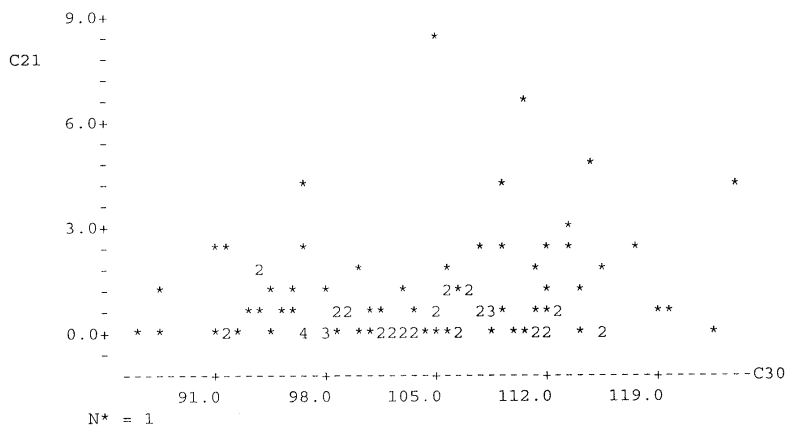
SOURCE	DF	SEQ SS
temp	1	5278.70
C4	1	2.51
C5	1	58.79
C6	1	612.63
C7	1	171.68
C8	1	2.02
sunday	1	66.60
holiday	1	411.85

Unusual Observations

Obs.	temp	consump	Fit	Stdev.Fit	Residual	St.Resid
17	45.9	128.674	114.713	1.921	13.961	2.23R
23	53.0	135.956	123.594	2.225	12.362	2.00R
30	42.3	96.705	109.401	1.746	-12.696	-2.01R
53	39.8	126.639	110.352	1.769	16.287	2.58R
87	32.0	108.911	96.326	1.920	12.585	2.01R
99	46.8	121.666	104.712	3.057	16.955	2.92R

R denotes an obs. with a large st. resid.

```
MTB > let c21=c20*c20
MTB > plot c21 c30
```



```
MTB > regress c21 1 c30
```

The regression equation is  
 $C21 = -1.87 + 0.0277 C30$

99 cases used 1 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	-1.873	1.786	-1.05	0.297
C30	0.02769	0.01711	1.62	0.109

s = 1.390      R-sq = 2.6%      R-sq(adj) = 1.6%

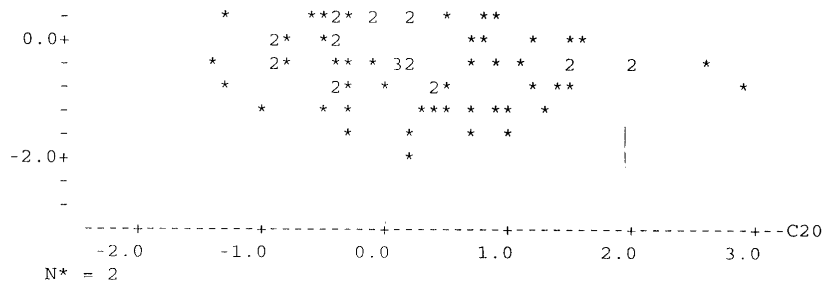
Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	5.064	5.064	2.62	0.109
Error	97	187.535	1.933		
Total	98	192.599			

```
MTB > lag c20 c21
```

```
MTB > plot c21 c20
```





MTB > corr c21 c20

Correlation of C21 and C20 = -0.391

```

MTB > let c22=c21-c20
MTB > let c23=c22*c22
MTB > let c24=c20*c20
MTB > let k1=sum(c23)
MTB > let k2=sum(c24)
MTB > let k3=k1/k2
MTB > print k3
K3          2.74540
MTB > lag c1 c31
MTB > lag c2 c32
MTB > lag c3 c33
MTB > lag c4 c34
MTB > lag c5 c35
MTB > lag c6 c35
MTB > lag c7 c37 ;
MTB > lag c8 c38
MTB > lag c9 c39
MTB > lag c10 c40
MTB > let c41=c1+0.391*c31
MTB > let c42=c2+0.391*c32
MTB > let c43=c3+0.391*c33
MTB > let c44=c4+0.391*c34
MTB > let c45=c5+0.391*c35
MTB > let c46=c6+0.391*c36
MTB > let c47=c7+0.391*c37
MTB > let c48=c8+0.391*c38
MTB > let c49=c9+0.391*c39
MTB > let c50=c10+0.391*c40
MTB > regress c41 8 c42 c44-c50 c20 c30

```

The regression equation is

$$C41 = 75.6 + 1.25 C42 + 2.31 C44 + 2.57 C45 + 5.12 C46 + 5.16 C47 - 3.31 C48 - 2.36 C49 - 7.70 C50$$

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	75.597	5.105	14.81	0.000
C42	1.24959	0.08115	15.40	0.000
C44	2.309	2.992	0.77	0.442

C45	2.573	2.309	1.11	0.268
C46	5.119	2.180	2.35	0.021
C47	5.156	3.297	1.56	0.121
C48	-3.312	2.137	-1.55	0.125
C49	-2.358	3.069	-0.77	0.444
C50	-7.697	2.104	-3.66	0.000

s = 6.033      R-sq = 74.8%      R-sq(adj) = 72.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	8	9603.0	1200.4	32.97	0.000
Error	89	3239.9	36.4		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4
C44	1	14.6
C45	1	182.8
C46	1	588.2
C47	1	554.7
C48	1	30.0
C49	1	20.2
C50	1	487.2

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
17	64.9	178.362	160.130	1.810	18.232	3.17R
23	68.9	176.654	164.015	1.971	12.639	2.22R
53	53.8	165.311	150.008	1.614	15.304	2.63R
99	64.5	158.944	144.543	2.963	14.400	2.74R

R denotes an obs. with a large st. resid.

MTB > regress c41 2 c42 c50

The regression equation is  
 $C41 = 78.9 + 1.22 C42 - 9.14 C50$

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	P
Constant	78.888	4.946	15.95	0.000
C42	1.22073	0.09042	13.50	0.000
C50	-9.145	2.284	-4.00	0.000

s = 6.789      R-sq = 65.9%      R-sq(adj) = 65.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	2	8464.0	4232.0	91.81	0.000
Error	95	4378.9	46.1		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4

C50 1 738.6

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
7	59.4	135.471	142.270	2.163	-6.799	-1.06 X
8	58.4	135.209	137.410	3.018	-2.201	-0.36 X
9	54.2	130.765	132.285	3.078	-1.521	-0.25 X
17	64.9	178.362	158.074	1.202	20.288	3.04R
23	68.9	176.654	163.009	1.513	13.645	2.06R
53	53.8	165.311	144.625	0.710	20.687	3.06R
98	63.8	141.048	147.675	2.199	-6.627	-1.03 X
99	64.5	158.944	144.882	3.015	14.061	2.31RX
100	67.5	148.467	148.581	3.051	-0.114	-0.02 X

R denotes an obs. with a large st. resid.  
 X denotes an obs. whose X value gives it large influence.

MTB > regress c41 5 c42 c46 c47 c48 c50

The regression equation is  
 $C41 = 76.3 + 1.26 C42 + 4.46 C46 + 5.27 C47 - 5.36 C48 - 8.35 C50$

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	76.284	4.650	16.41	0.000
C42	1.25596	0.08289	15.15	0.000
C46	4.460	1.357	3.29	0.001
C47	5.265	2.035	2.59	0.011
C48	-5.365	1.792	-2.99	0.004
C50	-8.348	2.120	-3.94	0.000

s = 6.173 R-sq = 72.7% R-sq(adj) = 71.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	9337.7	1867.5	49.02	0.000
Error	92	3505.2	38.1		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4
C46	1	368.6
C47	1	373.8
C48	1	279.1
C50	1	590.9

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
8	58.4	135.209	137.969	2.748	-2.761	-0.50 X
9	54.2	130.765	132.697	2.802	-1.932	-0.35 X
17	64.9	178.362	157.755	1.239	20.607	3.41R
23	68.9	176.654	162.832	1.482	13.821	2.31R
50	52.1	129.400	141.757	0.957	-12.356	-2.03R
53	53.8	165.311	150.121	1.650	15.190	2.55R
87	44.7	145.905	132.455	1.258	13.449	2.23R

99	64.5	158.944	145.657	2.747	13.286	2.40RX
100	67.5	148.467	149.463	2.782	-0.996	-0.18 X

R denotes an obs. with a large st. resid.  
X denotes an obs. whose X value gives it large influence.

MTB > regress c41 9 c42-c50;  
SUBC> noconstant.

The regression equation is  
 $C41 = 1.25 C42 + 53.1 C43 + 57.2 C44 + 56.7 C45 + 59.5 C46 + 80.8 C47$   
 $+ 42.7 C48 + 55.2 C49 - 7.70 C50$

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Noconstant				
C42	1.24959	0.08115	15.40	0.000
C43	53.073	3.584	14.81	0.000
C44	57.155	3.768	15.17	0.000
C45	56.725	3.604	15.74	0.000
C46	59.466	3.297	18.04	0.000
C47	80.829	4.609	17.54	0.000
C48	42.697	3.254	13.12	0.000
C49	55.250	3.813	14.49	0.000
C50	-7.697	2.104	-3.66	0.000

s = 6.033  
Analysis of Variance

SOURCE	DF	SS	MS	F	P
Regression	9	2063232	229248	6297.53	0.000
Error	89	3240	36		
Total	98	2066472			

SOURCE	DF	SEQ SS
C42	1	2048188
C43	1	216
C44	1	1
C45	1	508
C46	1	1343
C47	1	3316
C48	1	1172
C49	1	8002
C50	1	487

Unusual Observations						
Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
17	64.9	178.362	160.130	1.810	18.232	3.17R
23	68.9	176.654	164.015	1.971	12.639	2.22R
53	53.8	165.311	150.008	1.614	15.304	2.63R
99	64.5	158.944	144.543	2.963	14.400	2.74R

R denotes an obs. with a large st. resid.

MTB > regress c41 4 c42 c47 c48 c50

The regression equation is  
 $C41 = 79.1 + 1.23 C42 + 2.16 C47 - 6.04 C48 - 9.36 C50$

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	79.086	4.805	16.46	0.000
C42	1.23156	0.08680	14.19	0.000
C47	2.160	1.895	1.14	0.257
C48	-6.040	1.872	-3.23	0.002
C50	-9.355	2.205	-4.24	0.000

s = 6.490      R-sq = 69.5%      R-sq(adj) = 68.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	4	8926.2	2231.6	52.99	0.000
Error	93	3916.7	42.1		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4
C47	1	68.7
C48	1	374.3
C50	1	757.9

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
8	58.4	135.209	137.947	2.889	-2.738	-0.47 X
9	54.2	130.765	132.777	2.946	-2.012	-0.35 X
17	64.9	178.362	158.974	1.243	19.388	3.04R
23	68.9	176.654	163.952	1.517	12.701	2.01R
50	52.1	129.400	143.286	0.879	-13.886	-2.16R
53	53.8	165.311	145.405	0.857	19.906	3.09R
99	64.5	158.944	145.485	2.888	13.458	2.32RX
100	67.5	148.467	149.217	2.924	-0.749	-0.13 X

R denotes an obs. with a large st. resid.  
 X denotes an obs. whose X value gives it large influence.

MTB > print c1 c2 c3 c9 c10

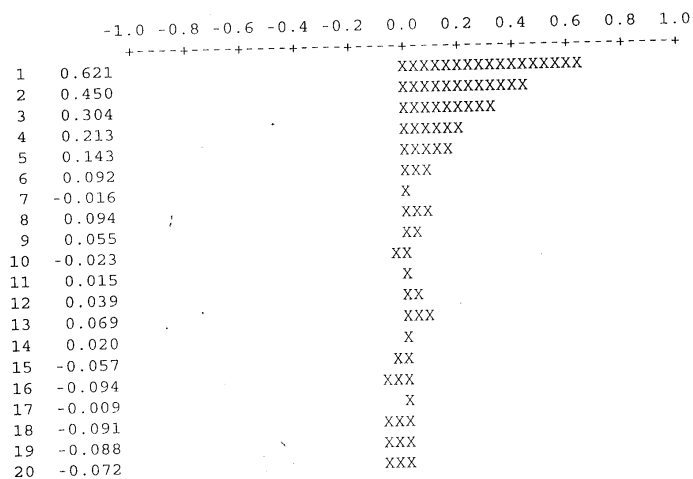
ROW	consump	temp	monday	sunday	holiday
1	*	36.7800	1	0	0
2	104.428	41.5540	0	0	0
3	103.743	35.9458	0	0	0
4	100.320	34.1945	0	0	0
5	109.285	43.9331	0	0	0
6	106.471	42.0398	0	0	0
7	93.841	42.9749	0	1	1
8	98.517	41.5569	1	0	1
9	92.245	37.9134	0	0	1
10	100.139	36.7285	0	0	0
11	113.020	40.7818	0	0	0
12	107.560	39.9786	0	0	0



13	109.513	40.8814	0	0	0
14	107.103	47.9436	0	1	0
15	106.859	43.6921	1	0	0
...					
91	85.780	28.1724	0	1	0
92	89.995	33.2663	1	0	0
93	100.446	33.5849	0	0	0
94	100.856	37.4965	0	0	0
95	123.543	46.2875	0	0	0
96	98.810	41.3563	0	0	0
97	116.906	47.7482	0	0	0
98	95.338	45.1705	0	1	1
99	121.666	46.8196	1	0	1
100	100.896	49.2046	0	0	1

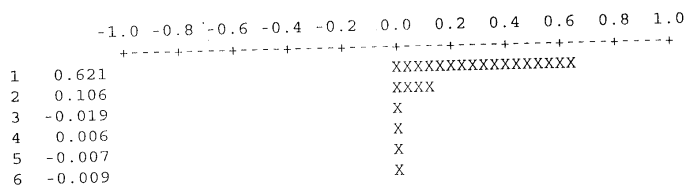
MTB > acf c2

ACF of temp



MTB > pacf c2

PACF of temp



7	-0.125	XXXX
8	0.234	XXXXXXXX
9	-0.073	XXX
10	-0.139	XXXX
11	0.124	XXXX
12	0.045	XX
13	0.026	XX
14	-0.133	XXXX
15	-0.026	XX
16	-0.064	XXX
17	0.113	XXXX
18	-0.107	XXXX
19	-0.009	X
20	0.042	XX

MTB > arima 1 0 0 c2

Estimates at each iteration

Iteration	SSE	Parameters	
0	3203.47	0.100	35.415
1	2714.14	0.250	29.518
2	2382.88	0.400	23.621
3	2209.68	0.550	17.722
4	2181.73	0.634	14.401
5	2181.64	0.639	14.215
6	2181.64	0.639	14.204

Relative change in each estimate less than 0.0010

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.6393	0.0795	8.04
Constant	14.2042	0.4720	30.09
Mean	39.378	1.309	

No. of obs.: 100

Residuals: SS = 2180.01 (backforecasts excluded)  
MS = 22.25 DF = 98

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	14.6 (DF=11)	37.2 (DF=23)	41.0 (DF=35)	49.4 (DF=47)

X