NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

July 2003 Supplementary Examination
3 Hours

This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.

Throughout this paper a_t represents white noise, $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A : Answer ALL questions from this section.

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(0,1,2) process and its differences.

[3 Marks]

2. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{l=1}^{p} \phi_\ell \rho_{k-\ell}.$$

Hence find the first 4 terms in the autocorrelation function for an AR(1) process with $\phi_1 = 0.2$.

[7 Marks]

3. (a) Derive the autocorrelation function for an ARIMA(0,0,1)x(0,0,1)$_{12}$ process.

[5 Marks]

(b) An ARIMA(0,0,0)x(0,0,1)$_{12}$ model is fitted to this process such that the sum of the squares of the residuals is minimised. Find the autocorrelation function for these residuals.

[4 Marks]

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4. Determine if the following process is stationary and/or invertible.

$$z_t - 2z_{t-1} = -0.4z_{t-1} + 0.1z_{t-2} + a_t - 0.6f_{t-1} - 0.2a_{t-1}$$

[3 Marks]

5. Describe how you would use an ordinary least squares method to obtain the best linear unbiased estimates of α and β for each of the following models,

   (a) $y_t = \alpha + \beta x_t + u_t$, where $u_t = x_t a_t$,

   (b) $y_t = \alpha {x_t}^{\beta} \exp u_t$, where $u_t = x_t^2 a_t$.

[7 Marks]

6. Comment on the effect on the estimation of parameters in a regression model of autocorrelated residuals of lag one. Describe a technique for dealing with this violation of the assumptions of the ordinary least squares model.

[5 Marks]

SECTION B : Answer TWO questions from this section.
Each question carries 33 marks.

7. (a) Write an ARIMA(1,0,1) model in general linear process form,

$$z_t - \mu = \sum_{j=0}^{\infty} \psi_j a_{t-j}.$$

[6 Marks]

   (b) Show that, if a stationary model is written in general linear process form, then the covariance $E((z_t - \mu)(z_{t-k} - \mu))$ can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell}\psi_{\ell}\sigma^2.$$

[4 Marks]

(c) Hence find the autocorrelation function for an ARIMA(1,0,1) process.

[6 Marks]

(d) The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^{k} \phi_{k\ell}\rho_{j-\ell} \qquad j = 1, 2, \ldots, k.$$

Use these equations to find the partial autocorrelation function, ϕ_{kk}, for an ARIMA(1,0,1) process, up to lag 3, given that $\theta_1 = 0$, and $\phi_1 = 0.3$.

[9 Marks]

(e) Describe the autocorrelation and partial autocorrelation functions you would expect from a time series of 400 observations and its differences from an ARIMA(1,2,1), noconstant, process with $\theta_1 = 0$ and $\phi_1 = 0.3$.

[3 Marks]

(f) Given the following values from a time series following an ARIMA(1,2,1), noconstant, model with $\theta_1 = 0$ and $\phi_1 = 0.3$, calculate forecasts for the next 2 periods.

t	z_t	f_t
398	17.84	17.67
399	17.73	16.87
400	18.39	17.92

[5 Marks]

8. An operations research consultant has been asked to analyse daily data on the traffic intensity levels in a particular city. The purpose is to produce a model so that forecasts can be produced for traffic intensity. The consultant used the MINITAB statistical package to produce the output given in appendix A.

(a) Explain clearly the output given, including

i. the reasons for using each of the commands,

ii. a summary of the conclusions of the output from each command.

[25 Marks]

(b) Using the final model used by the consultant, calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the city council.

[8 Marks]

9. A statistical consultant has been asked to analyse daily data on electricity consumption in a particular city in the North Africa. The purpose is to produce a model so that forecasts can be produced for electricity consumption on a particular day with an estimated temperature. The consultant used the MINITAB statistical package to produce the output given in appendix B.

(a) Explain clearly the output given, including

   i. the reasons for using each of the commands,

   ii. a summary of the conclusions of the output from each command.

[25 Marks]

(b) On the basis of the information given, specify a model which you think is most appropriate for these data. Use this model to calculate forecasts for the next two days. Describe any further tests you would carry out before finally recommending a model to the electricity authority.

[8 Marks]

END OF EXAMINATION PAPER

Table of Laplace Transforms

$F(s)$	$f(t)$
$e^{-cs}/s, c > 0$	$H(t-c)$
$e^{-cs}F(s)$	$f(t-c)H(t-c)$
$1/s$	1
$1/s^{n+1}$	$t^n/n!$
$1/(s+a)$	e^{-at}
$1/(s+a)^{n+1}$	$t^n e^{-at}/n!$
$\dfrac{k}{s^2+k^2}$	$\sin kt$
$\dfrac{s}{s^2+k^2}$	$\cos kt$
$\dfrac{k}{s^2-k^2}$	$\sinh kt$
$\dfrac{s}{s^2-k^2}$	$\cosh kt$
$\dfrac{1}{(s^2+k^2)^2}$	$\dfrac{1}{2k^3}[\sin kt - kt\cos kt]$
$\dfrac{s}{(s^2+k^2)^2}$	$\dfrac{1}{2k}t\sin kt$
$\ln(1+1/s)$	$(1-e^{-t})/t$
$\ln[(s+k)/(s-k)]$	$2(\sinh kt)/t$
$\ln(1-k^2/s^2)$	$2(1-\cosh kt)/t$
$\ln(1+k^2/s^2)$	$2(1-\cos kt)/t$
$\tan^{-1}(k/s)$	$\sin kt/t$

Appendix A

```
MTB> name c1 'traff'

MTB > tsplot c1

traff    -
         -
         -
    60.0+
         -
         -
         -
         -
    40.0+
         -
         -
         -
         -
    20.0+                           8      45       23 5 7 9  2 456789
         - 1234567890123456789012345 67 90123   678901  4 6 8 01 3      0
         -
         -
         +---------+---------+---------+---------+---------+---------+
         0        10        20        30        40        50        60

. . .

traff    -
         -
         -
    60.0+
         -                                                  8      5
         -                                           1
         -     6      3      0      7      4                   1    6 8
         -                             0      7      4      9  2       9
    40.0+ 2     7 9   4 6    1 3    8  1   5  8 0 2  5          34      0
         -  345    012    789    456    23      9      67  0      7
         - 1      8      5              9     6     3
         -                      2
         -
    20.0+
         -
         -
         -
         +---------+---------+---------+---------+---------+---------+
       240       250       260       270       280       290       300
```

```
traff   -
        -
        -                                                          8
    60.0+                                     7      4      1  4
        -  2                    3      0         0    5 7    2      9
        -         9      6  9      6    1 3    8         8      5
        -    5      2    7                4      1
        -  3  6   0  3      0   4  7        6     23     90     67
    40.0+        8     45     12     89     5   9      6      3      0
        - 1  4  7   1      8      5      2
        -
        -
        -
    20.0+
        -
        -
        -
         +---------+---------+---------+---------+---------+---------+
       300       310       320       330       340       350       360
```

```
traff   -                                    6
        -     5  8                 6  9
        -            2  5   9  2          3
    60.0+ 1    6      3      0      7
        -                              0   4  7
        -  2      9      6      3
        -    4      1      8            12     8
        -   3   7  0   4  7   1   5  8      5
    40.0+                        4
        -
        -
        -
        -
    20.0+
        -
        -
        -
         +---------+---------+---------+---------+
       360       370       380       390       400
```

```
MTB > acf c1

ACF of traff

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1   0.841                          XXXXXXXXXXXXXXXXXXXXXX
  2   0.796                          XXXXXXXXXXXXXXXXXXXXX
  3   0.889                          XXXXXXXXXXXXXXXXXXXXXXX
  4   0.881                          XXXXXXXXXXXXXXXXXXXXXXX
  5   0.775                          XXXXXXXXXXXXXXXXXXXX
  6   0.807                          XXXXXXXXXXXXXXXXXXXXX
```

```
     7   0.948                           XXXXXXXXXXXXXXXXXXXXXXXXX
     8   0.798                           XXXXXXXXXXXXXXXXXXXXX
     9   0.758                           XXXXXXXXXXXXXXXXXXXX
    10   0.847                           XXXXXXXXXXXXXXXXXXXXXX
    11   0.835                           XXXXXXXXXXXXXXXXXXXXXX
    12   0.733                           XXXXXXXXXXXXXXXXXXX
    13   0.765                           XXXXXXXXXXXXXXXXXXXX
    14   0.897                           XXXXXXXXXXXXXXXXXXXXXXX
    15   0.756                           XXXXXXXXXXXXXXXXXXXX
    16   0.721                           XXXXXXXXXXXXXXXXXXX
    17   0.807                           XXXXXXXXXXXXXXXXXXXXX
    18   0.792                           XXXXXXXXXXXXXXXXXXXXX
    19   0.695                           XXXXXXXXXXXXXXXXXX
    20   0.725                           XXXXXXXXXXXXXXXXXXX
    21   0.851                           XXXXXXXXXXXXXXXXXXXXXX
    22   0.715                           XXXXXXXXXXXXXXXXXXX
    23   0.683                           XXXXXXXXXXXXXXXXXX
    24   0.766                           XXXXXXXXXXXXXXXXXXXX
    25   0.748                           XXXXXXXXXXXXXXXXXXXX
    26   0.654                           XXXXXXXXXXXXXXXXX
    27   0.683                           XXXXXXXXXXXXXXXXXX
    28   0.803                           XXXXXXXXXXXXXXXXXXXXX
    29   0.672                           XXXXXXXXXXXXXXXXXX

MTB > diff c1 c2
MTB > acf c2

ACF of C2
               -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
     1  -0.373                  XXXXXXXXXX
     2  -0.455                XXXXXXXXXXXX
     3   0.345                           XXXXXXXXXX
     4   0.304                           XXXXXXXXX
     5  -0.450                XXXXXXXXXXXX
     6  -0.338                   XXXXXXXXX
     7   0.938                           XXXXXXXXXXXXXXXXXXXXXXXX
     8  -0.359                  XXXXXXXXXX
     9  -0.428                XXXXXXXXXXXX
    10   0.346                           XXXXXXXXXX
    11   0.278                           XXXXXXXX
    12  -0.437                XXXXXXXXXXXX
    13  -0.311                   XXXXXXXXX
    14   0.886                           XXXXXXXXXXXXXXXXXXXXXXX
    15  -0.349                  XXXXXXXXXX
    16  -0.397                 XXXXXXXXXXX
    17   0.335                           XXXXXXXXX
    18   0.256                           XXXXXXX
    19  -0.414                 XXXXXXXXXXX
    20  -0.299                    XXXXXXXX
    21   0.848                           XXXXXXXXXXXXXXXXXXXXXX
    22  -0.339                   XXXXXXXXX
    23  -0.381                 XXXXXXXXXXX
    24   0.338                           XXXXXXXXX
    25   0.234                           XXXXXXX
    26  -0.399                 XXXXXXXXXXX
    27  -0.279                    XXXXXXXX
```

```
 28    0.808                          XXXXXXXXXXXXXXXXXXXXX
 29   -0.326                  XXXXXXXXX

MTB > diff 7 c1 c3
MTB > acf c3

ACF of C3

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1    0.326                          XXXXXXXXX
  2    0.159                          XXXXX
  3    0.178                          XXXXX
  4    0.010                          X
  5    0.003                          X
  6    0.007                          X
  7   -0.179                      XXXXX
  8   -0.057                         XX
  9   -0.010                          X
 10    0.007                          X
 11   -0.043                         XX
 12   -0.075                        XXX
 13    0.008                          X
 14   -0.045                         XX
 15   -0.086                        XXX
 16   -0.087                        XXX
 17   -0.070                        XXX
 18   -0.032                         XX
 19    0.017                          X
 20   -0.007                          X
 21    0.071                          XXX
 22    0.098                          XXX
 23    0.168                          XXXXX
 24    0.175                          XXXXX
 25    0.118                          XXXX
 26    0.038                          XX
 27    0.007                          X
 28    0.011                          X
 29   -0.036                         XX

MTB > pacf c3

PACF of C3

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1    0.326                          XXXXXXXXX
  2    0.058                          XX
  3    0.124                          XXXX
  4   -0.100                        XXX
  5    0.005                          X
  6   -0.008                          X
  7   -0.189                     XXXXXX
  8    0.060                          XXX
  9    0.018                          X
 10    0.070                          XXX
 11   -0.095                        XXX
```

```
 12  -0.059                        XX
 13   0.068                         XXX
 14  -0.088                       XXX
 15  -0.046                        XX
 16  -0.065                       XXX
 17   0.033                         XX
 18  -0.005                         X
 19   0.012                         X
 20   0.009                         X
 21   0.074                         XXX
 22   0.042                         XX
 23   0.100                         XXX
 24   0.079                         XXX
 25   0.022                         XX
 26  -0.060                       XXX
 27  -0.047                        XX
 28   0.039                         XX
 29  -0.036                        XX

MTB > arima 1 0 0 0 1 1 7 c1 c20 c30;
SUBC> const.

Estimates at each iteration
Iteration        SSE     Parameters
    0         672.826     0.100     0.100     0.732
    1         628.362     0.250     0.176     0.570
    2         620.658     0.337     0.227     0.477
    3         620.642     0.341     0.227     0.471
    4         620.642     0.341     0.227     0.471
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type       Estimate     St. Dev.  t-ratio
AR   1       0.3410      0.0479      7.12
SMA  7       0.2269      0.0509      4.46
Constant    0.47088     0.04947      9.52

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.:  Original series 398, after differencing 391
Residuals:    SS = 620.473  (backforecasts excluded)
              MS =   1.599  DF = 388

Modified Box-Pierce chisquare statistic
Lag                 12             24             36             48
Chisquare   19.7(DF=10)   33.8(DF=22)   47.6(DF=34)   57.5(DF=46)

MTB > let c21=c20*c20
MTB > regr c21 1 c30

The regression equation is
C21 = - 1.83 + 0.104 C30

391 cases used 7 cases contain missing values

Predictor        Coef        Stdev    t-ratio        p
Constant      -1.8257       0.3876      -4.71    0.000
C30           0.10385      0.01097       9.46    0.000
```

```
s = 2.814        R-sq = 18.7%      R-sq(adj) = 18.5%

Analysis of Variance

SOURCE         DF          SS          MS          F        p
Regression      1      709.05      709.05      89.57    0.000
Error         389     3079.52        7.92
Total         390     3788.57

MTB > let c10=loge(c1)
MTB > acf c10

ACF of c10

            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
  1   0.841                            XXXXXXXXXXXXXXXXXXXXXX
  2   0.796                            XXXXXXXXXXXXXXXXXXXXX
  3   0.889                            XXXXXXXXXXXXXXXXXXXXXXX
  4   0.881                            XXXXXXXXXXXXXXXXXXXXXXX
  5   0.775                            XXXXXXXXXXXXXXXXXXXX
  6   0.807                            XXXXXXXXXXXXXXXXXXXXX
  7   0.948                            XXXXXXXXXXXXXXXXXXXXXXXXX
  8   0.798                            XXXXXXXXXXXXXXXXXXXXX
  9   0.758                            XXXXXXXXXXXXXXXXXXXX
 10   0.847                            XXXXXXXXXXXXXXXXXXXXXX
 11   0.835                            XXXXXXXXXXXXXXXXXXXXXX
 12   0.733                            XXXXXXXXXXXXXXXXXXX
 13   0.765                            XXXXXXXXXXXXXXXXXXXX
 14   0.897                            XXXXXXXXXXXXXXXXXXXXXXX
 15   0.756                            XXXXXXXXXXXXXXXXXXXX
 16   0.721                            XXXXXXXXXXXXXXXXXXX
 17   0.807                            XXXXXXXXXXXXXXXXXXXXX
 18   0.792                            XXXXXXXXXXXXXXXXXXXXX
 19   0.695                            XXXXXXXXXXXXXXXXXX
 20   0.725                            XXXXXXXXXXXXXXXXXXX
 21   0.851                            XXXXXXXXXXXXXXXXXXXXXX
 22   0.715                            XXXXXXXXXXXXXXXXXXX
 23   0.683                            XXXXXXXXXXXXXXXXXX
 24   0.766                            XXXXXXXXXXXXXXXXXXXX
 25   0.748                            XXXXXXXXXXXXXXXXXXXX
 26   0.654                            XXXXXXXXXXXXXXXXX
 27   0.683                            XXXXXXXXXXXXXXXXXX
 28   0.803                            XXXXXXXXXXXXXXXXXXXXX
 29   0.672                            XXXXXXXXXXXXXXXXXX

MTB > diff 7 c10 c11
MTB > acf c11

ACF of C11

            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
  1   0.355                            XXXXXXXXXX
  2   0.165                            XXXXX
  3   0.107                            XXXX
```

```
  4   -0.020                          XX
  5   -0.036                          XX
  6   -0.085                         XXX
  7   -0.277                    XXXXXXXX
  8   -0.133                        XXXX
  9   -0.063                         XXX
 10   -0.002                           X
 11   -0.003                           X
 12   -0.056                          XX
 13   -0.006                           X
 14   -0.030                          XX
 15   -0.054                          XX
 16   -0.058                          XX
 17   -0.079                         XXX
 18   -0.062                         XXX
 19    0.041                           XX
 20    0.023                           XX
 21    0.064                           XXX
 22    0.066                           XXX
 23    0.116                           XXXX
 24    0.170                           XXXXX
 25    0.069                           XXX
 26    0.005                           X
 27   -0.036                          XX
 28   -0.054                          XX
 29   -0.046                          XX
```

```
MTB > pacf c11
```

PACF of C11

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1    0.355                          XXXXXXXXXX
  2    0.045                          XX
  3    0.041                          XX
  4   -0.087                        XXX
  5   -0.016                          X
  6   -0.070                        XXX
  7   -0.248                    XXXXXXX
  8    0.054                          XX
  9    0.020                          X
 10    0.060                          XXX
 11   -0.046                         XX
 12   -0.075                        XXX
 13    0.016                          X
 14   -0.104                       XXXX
 15   -0.033                         XX
 16   -0.036                         XX
 17   -0.017                          X
 18   -0.027                         XX
 19    0.054                          XX
 20   -0.002                          X
 21    0.023                          XX
 22   -0.001                          X
 23    0.075                          XXX
 24    0.096                          XXX
```

```
  25  -0.059
  26  -0.014                                XX
  27  -0.047                                 X
  28   0.005                                XX
  29  -0.019                                 X
                                             X

MTB > arima 1 0 0 0 1 1 7 c10 c20 c30;
SUBC> const.

Estimates at each iteration
Iteration       SSE        Parameters
    0        4.40891      0.100     0.100     0.110
    1        1.21445      0.250     0.181     0.053
    2        0.46034      0.346     0.285     0.016
    3        0.45902      0.345     0.301     0.015
    4        0.45900      0.343     0.295     0.015
    5        0.45900      0.344     0.295     0.015
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type        Estimate      St. Dev.   t-ratio
AR    1       0.3436       0.0477      7.21
SMA   7       0.2953       0.0490      6.02
Constant    0.014595     0.001233     11.84

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.:  Original series 398, after differencing 391
Residuals:    SS =  0.458926  (backforecasts excluded)
              MS =  0.001183  DF = 388

Modified Box-Pierce chisquare statistic
Lag                12             24             36             48
Chisquare     7.6(DF=10)    21.6(DF=22)    30.3(DF=34)    38.6(DF=46)

MTB > plot c20 c30

C20      -
         -                 *                       *                 *
         -              *       *
  0.070+          **         2 *     *       **                *
         -      *                        *  **     * * ** 2   *
         -    **   ******2* **        2*** 2    ***2* 3     **2      ** *
         -     **22  2 2*3**  23     **22 * 2*22 2 *     2* *  *
         -      22**3222**2** 32**223* ***2 *   **2**    2 22    *
  0.000+    *2*2*3 32*2 *3*2** 3 ***2**423** *2*32  2  ***
         -     *23424* 2**** *   3** *2 *2 3*233323**3*    4 *
         -     ** **4* 2*2*2***   * ***    ** ***32** ** **   * **
         -          2*** **2**2       **   * * 22***   * *   *   *   *
         -      *   *  2 **     *       **    **    *  * *   *   *   **
 -0.070+         *   2 *    *                      2*   **
         -                              *
         -                                                  *
         -
          --+---------+---------+---------+---------+---------+----C30
          2.70      3.00      3.30      3.60      3.90      4.20
N* = 7
```

```
MTB > acf c20

ACF of C20

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1  -0.014                          X
  2   0.018                          X
  3   0.085                          XXX
  4  -0.031                         XX
  5  -0.016                          X
  6   0.017                          X
  7  -0.001                          X
  8  -0.057                         XX
  9  -0.025                         XX
 10   0.049                          XX
 11  -0.006                          X
 12  -0.059                         XX
 13   0.026                          XX
 14  -0.005                          X
 15  -0.052                         XX
 16  -0.021                         XX
 17  -0.006                          X
 18  -0.072                        XXX
 19   0.052                          XX
 20  -0.010                          X
 21   0.041                          XX
 22  -0.005                          X
 23   0.036                          XX
 24   0.137                          XXXX
 25  -0.025                         XX
 26  -0.003                          X
 27  -0.016                          X
 28  -0.014                          X
 29  -0.014                          X

MTB > cdf 7.6;
SUBC> chisq 10.
    7.6000     0.3322
MTB > cdf 21.6;
SUBC> chisq 22.
   21.6000     0.5160
MTB > pacf c20

PACF of C20

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1  -0.014                          X
  2   0.017                          X
  3   0.085                          XXX
  4  -0.029                         XX
  5  -0.021                         XX
  6   0.010                          X
  7   0.005                          X
```

```
  8   -0.055                          XX
  9   -0.031                          XX
 10    0.052                           XX
 11    0.007                           X
 12   -0.061                         XXX
 13    0.012                           X
 14    0.003                           X
 15   -0.042                          XX
 16   -0.035                          XX
 17   -0.007                           X
 18   -0.057                          XX
 19    0.054                           XX
 20   -0.018                           X
 21    0.049                           XX
 22   -0.009                           X
 23    0.032                           XX
 24    0.128                           XXXX
 25   -0.018                           X
 26   -0.021                          XX
 27   -0.039                          XX
 28    0.006                           X
 29   -0.011                           X


MTB > arima 1 0 1 0 1 1 7 c10 c20 c30;
SUBC> const.

Estimates at each iteration
Iteration        SSE      Parameters
    0         5.35331    0.100    0.100     0.100    0.110
    1         0.77064    0.250   -0.050     0.215    0.040
    2         0.61559    0.400    0.070     0.221    0.028
    3         0.45706    0.475    0.143     0.293    0.012
    4         0.45699    0.477    0.151     0.301    0.012
    5         0.45699    0.478    0.152     0.301    0.012
    6         0.45699    0.478    0.153     0.302    0.012
    7         0.45699    0.478    0.153     0.302    0.012
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type        Estimate      St. Dev.  t-ratio
AR   1        0.4779       0.1274      3.75
MA   1        0.1529       0.1433      1.07
SMA  7        0.3016       0.0490      6.15
Constant    0.011608     0.001028     11.29

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.:  Original series 398, after differencing 391
Residuals:    SS = 0.456056  (backforecasts excluded)
              MS = 0.001178  DF = 387

Modified Box-Pierce chisquare statistic
Lag                  12            24             36             48
Chisquare     7.1(DF= 9)    20.7(DF=21)    29.4(DF=33)    37.6(DF=45)

MTB > arima 1 0 0 0 1 1 7 c10 c20 c30;
SUBC> const.
```

```
Final Estimates of Parameters
Type        Estimate      St. Dev.   t-ratio
AR   1        0.3436        0.0477      7.21
SMA  7        0.2953        0.0490      6.02
Constant  0.014595      0.001233     11.84

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.:  Original series 398, after differencing 391
Residuals:    SS = 0.458926  (backforecasts excluded)
              MS = 0.001183  DF = 388


Modified Box-Pierce chisquare statistic
Lag                  12              24              36              48
Chisquare    7.6(DF=10)    21.6(DF=22)    30.3(DF=34)    38.6(DF=46)

MTB > print c10 c20 c30

 ROW       C10          C20         C30

   1   2.73925            *           *
   2   2.77592            *           *
   3   2.74592            *           *
   4   2.78259            *           *
   5   2.75259            *           *
   6   2.75592            *           *
   7   2.79259            *           *
   8   2.76259    0.0040161     2.75857
   9   2.85783    0.0592978     2.79853

 ...

 384   3.72841   -0.0683545     3.79676
 385   3.81829   -0.0494174     3.86771
 386   4.22086    0.0476971     4.17317
 387   4.07244   -0.0505411     4.12298
 388   3.77334   -0.0006607     3.77400
 389   4.24110    0.0607415     4.18036
 390   4.01632    0.0165476     3.99977
 391   3.84007    0.0560169     3.78406
 392   3.86210   -0.0237489     3.88585
 393   4.18091   -0.0555130     4.23642
 394   4.04642   -0.0418117     4.08823
 395   3.77660   -0.0025844     3.77919
 396   4.27755    0.0386729     4.23888
 397   4.04619    0.0076406     4.03855
 398   3.85115    0.0027657     3.84839
```

APPENDIX B

```
MTB > name c1 'consump' c2 'temp' c10 'holiday'
MTB > name c3 'day'
MTB > set c3
DATA> 15(1:7)
DATA> end
MTB > delete 101 102 103 104 105 c3
MTB > indicators c3 c3-c9
MTB > name c3 'monday' c9 'sunday'
MTB > regress c1 9 c2-c10

*   sunday is highly correlated with other X variables
*   sunday has been removed from the equation


The regression equation is
consump = 48.7 + 1.32 temp + 3.09 monday + 4.73 C4 + 5.21 C5 + 9.02 C6
          + 4.60 C7 + 0.75 C8 - 9.05 holiday

99 cases used 1 cases contain missing values
```

Predictor	Coef	Stdev	t-ratio	p
Constant	48.730	4.794	10.17	0.000
temp	1.3229	0.1128	11.73	0.000
monday	3.093	2.479	1.25	0.216
C4	4.730	2.442	1.94	0.056
C5	5.212	2.514	2.07	0.041
C6	9.016	2.521	3.58	0.001
C7	4.599	2.520	1.83	0.071
C8	0.753	2.513	0.30	0.765
holiday	-9.050	2.925	-3.09	0.003

s = 6.560 R-sq = 63.0% R-sq(adj) = 59.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	6604.78	825.60	19.19	0.000
Error	90	3872.88	43.03		
Total	98	10477.66			

SOURCE	DF	SEQ SS
temp	1	5278.70
monday	1	37.28
C4	1	0.25
C5	1	43.75
C6	1	595.42
C7	1	209.31
C8	1	28.22
holiday	1	411.85

Unusual Observations

Obs.	temp	consump	Fit	Stdev.Fit	Residual	St.Resid
17	45.9	128.674	114.713	1.921	13.961	2.23R
23	53.0	135.956	123.594	2.225	12.362	2.00R
30	42.3	96.705	109.401	1.746	-12.696	-2.01R

53	39.8	126.639	110.352	1.769	16.287	2.58R
87	32.0	108.911	96.326	1.920	12.585	2.01R
99	46.8	121.666	104.712	3.057	16.955	2.92R

R denotes an obs. with a large st. resid.

MTB > regress c1 8 c2 c4-c10 c20 c30

The regression equation is
consump = 51.8 + 1.32 temp + 1.64 C4 + 2.12 C5 + 5.92 C6 + 1.51 C7 - 2.34 C8
- 3.09 sunday - 9.05 holiday

99 cases used 1 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	51.823	4.779	10.84	0.000
temp	1.3229	0.1128	11.73	0.000
C4	1.637	2.443	0.67	0.504
C5	2.119	2.514	0.84	0.401
C6	5.923	2.520	2.35	0.021
C7	1.507	2.519	0.60	0.551
C8	-2.340	2.513	-0.93	0.354
sunday	-3.093	2.479	-1.25	0.216
holiday	-9.050	2.925	-3.09	0.003

s = 6.560   R-sq = 63.0%   R-sq(adj) = 59.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	6604.78	825.60	19.19	0.000
Error	90	3872.88	43.03		
Total	98	10477.66			

SOURCE	DF	SEQ SS
temp	1	5278.70
C4	1	2.51
C5	1	58.79
C6	1	612.63
C7	1	171.68
C8	1	2.02
sunday	1	66.60
holiday	1	411.85

Unusual Observations

Obs.	temp	consump	Fit	Stdev.Fit	Residual	St.Resid
17	45.9	128.674	114.713	1.921	13.961	2.23R
23	53.0	135.956	123.594	2.225	12.362	2.00R
30	42.3	96.705	109.401	1.746	-12.696	-2.01R
53	39.8	126.639	110.352	1.769	16.287	2.58R
87	32.0	108.911	96.326	1.920	12.585	2.01R
99	46.8	121.666	104.712	3.057	16.955	2.92R

R denotes an obs. with a large st. resid.

```
MTB > let c21=c20*c20
MTB > plot c21 c30

      9.0+
         -                             *
 C21     -
         -
         -
      6.0+                                     *
         -
         -
         -                 *                 *                     *
         -
      3.0+                                          *
         -         **      *              * *    * *     *
         -             2       *       *       *     *
         -    *         * *  *      *   2*2     *  *
         -            ** **   22 **  * 2   23*  **2        **
      0.0+  * *    *2*  *  4 3* **2222***2  * **22  * 2           *
         -
           --------+---------+---------+---------+---------+--------C30
                 91.0      98.0     105.0     112.0     119.0
       N* = 1

MTB > regress c21 1 c30
```

The regression equation is
C21 = - 1.87 + 0.0277 C30

99 cases used 1 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	-1.873	1.786	-1.05	0.297
C30	0.02769	0.01711	1.62	0.109

s = 1.390 R-sq = 2.6% R-sq(adj) = 1.6%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	5.064	5.064	2.62	0.109
Error	97	187.535	1.933		
Total	98	192.599			

```
MTB > lag c20 c21

MTB > plot c21 c20

         -
 C21     -
         -          *
         -        *                 *
      2.0+         *               *
         -             *   * *     2                      *
         -             2*      **
         -     *    ** ** *2*    *          *
```

```
      -              *        **2* 2  2  *  **
  0.0+                   2*  *2            **   *  **
      -             *    2*    ** * 32    * * *   2    2     *
      -              *          2*  *    2*       * **            *
      -                 *    * *      *** * **  *
      -                        *    *    *  *         |
 -2.0+                              *                 !
      -
      -
        ----+---------+---------+---------+---------+---------+--C20
          -2.0      -1.0       0.0       1.0       2.0       3.0
    N* = 2

MTB > corr c21 c20

Correlation of C21 and C20 = -0.391

MTB > let c22=c21-c20
MTB > let c23=c22*c22
MTB > let c24=c20*c20
MTB > let k1=sum(c23)
MTB > let k2=sum(c24)
MTB > let k3=k1/k2
MTB > print k3
K3       2.74540
MTB > lag c1 c31
MTB > lag c2 c32
MTB > lag c3 c33
MTB > lag c4 c34
MTB > lag c5 c35
MTB > lag c6 c35
MTB > lag c7 c37
MTB > lag c8 c38
MTB > lag c9 c39
MTB > lag c10 c40
MTB > let c41=c1+0.391*c31
MTB > let c42=c2+0.391*c32
MTB > let c43=c3+0.391*c33
MTB > let c44=c4+0.391*c34
MTB > let c45=c5+0.391*c35
MTB > let c46=c6+0.391*c36
MTB > let c47=c7+0.391*c37
MTB > let c48=c8+0.391*c38
MTB > let c49=c9+0.391*c39
MTB > let c50=c10+0.391*c40
MTB > regress c41 8 c42 c44-c50 c20 c30

The regression equation is
C41 = 75.6 + 1.25 C42 + 2.31 C44 + 2.57 C45 + 5.12 C46 + 5.16 C47 - 3.31 C48
           - 2.36 C49 - 7.70 C50

98 cases used 2 cases contain missing values

Predictor        Coef       Stdev     t-ratio        p
Constant       75.597       5.105       14.81    0.000
C42           1.24959     0.08115       15.40    0.000
C44             2.309       2.992        0.77    0.442
```

C45	2.573	2.309	1.11	0.268
C46	5.119	2.180	2.35	0.021
C47	5.156	3.297	1.56	0.121
C48	-3.312	2.137	-1.55	0.125
C49	-2.358	3.069	-0.77	0.444
C50	-7.697	2.104	-3.66	0.000

s = 6.033   R-sq = 74.8%   R-sq(adj) = 72.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	8	9603.0	1200.4	32.97	0.000
Error	89	3239.9	36.4		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4
C44	1	14.6
C45	1	182.8
C46	1	588.2
C47	1	554.7
C48	1	30.0
C49	1	20.2
C50	1	487.2

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
17	64.9	178.362	160.130	1.810	18.232	3.17R
23	68.9	176.654	164.015	1.971	12.639	2.22R
53	53.8	165.311	150.008	1.614	15.304	2.63R
99	64.5	158.944	144.543	2.963	14.400	2.74R

R denotes an obs. with a large st. resid.

MTB > regress c41 2 c42 c50

The regression equation is
C41 = 78.9 + 1.22 C42 - 9.14 C50

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	78.888	4.946	15.95	0.000
C42	1.22073	0.09042	13.50	0.000
C50	-9.145	2.284	-4.00	0.000

s = 6.789   R-sq = 65.9%   R-sq(adj) = 65.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	8464.0	4232.0	91.81	0.000
Error	95	4378.9	46.1		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4

C50	1	738.6

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
7	59.4	135.471	142.270	2.163	-6.799	-1.06 X
8	58.4	135.209	137.410	3.018	-2.201	-0.36 X
9	54.2	130.765	132.285	3.078	-1.521	-0.25 X
17	64.9	178.362	158.074	1.202	20.288	3.04R
23	68.9	176.654	163.009	1.513	13.645	2.06R
53	53.8	165.311	144.625	0.710	20.687	3.06R
98	63.8	141.048	147.675	2.199	-6.627	-1.03 X
99	64.5	158.944	144.882	3.015	14.061	2.31RX
100	67.5	148.467	148.581	3.051	-0.114	-0.02 X

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

```
MTB > regress c41 5 c42 c46 c47 c48 c50
```

The regression equation is
C41 = 76.3 + 1.26 C42 + 4.46 C46 + 5.27 C47 - 5.36 C48 - 8.35 C50

98 cases used 2 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	76.284	4.650	16.41	0.000
C42	1.25596	0.08289	15.15	0.000
C46	4.460	1.357	3.29	0.001
C47	5.265	2.035	2.59	0.011
C48	-5.365	1.792	-2.99	0.004
C50	-8.348	2.120	-3.94	0.000

s = 6.173 R-sq = 72.7% R-sq(adj) = 71.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	9337.7	1867.5	49.02	0.000
Error	92	3505.2	38.1		
Total	97	12842.9			

SOURCE	DF	SEQ SS
C42	1	7725.4
C46	1	368.6
C47	1	373.8
C48	1	279.1
C50	1	590.9

Unusual Observations

Obs.	C42	C41	Fit	Stdev.Fit	Residual	St.Resid
8	58.4	135.209	137.969	2.748	-2.761	-0.50 X
9	54.2	130.765	132.697	2.802	-1.932	-0.35 X
17	64.9	178.362	157.755	1.239	20.607	3.41R
23	68.9	176.654	162.832	1.482	13.821	2.31R
50	52.1	129.400	141.757	0.957	-12.356	-2.03R
53	53.8	165.311	150.121	1.650	15.190	2.55R
87	44.7	145.905	132.455	1.258	13.449	2.23R

```
 99      64.5    158.944    145.657      2.747     13.286      2.40RX
100      67.5    148.467    149.463      2.782     -0.996     -0.18 X

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

MTB > regress c41 9 c42-c50;
SUBC> noconstant.

The regression equation is
C41 = 1.25 C42 + 53.1 C43 + 57.2 C44 + 56.7 C45 + 59.5 C46 + 80.8 C47
         + 42.7 C48 + 55.2 C49 - 7.70 C50

98 cases used 2 cases contain missing values

Predictor       Coef       Stdev    t-ratio       p
Noconstant
C42          1.24959     0.08115      15.40   0.000
C43           53.073       3.584      14.81   0.000
C44           57.155       3.768      15.17   0.000
C45           56.725       3.604      15.74   0.000
C46           59.466       3.297      18.04   0.000
C47           80.829       4.609      17.54   0.000
C48           42.697       3.254      13.12   0.000
C49           55.250       3.813      14.49   0.000
C50           -7.697       2.104      -3.66   0.000

s = 6.033
Analysis of Variance

SOURCE       DF          SS          MS        F        p
Regression    9     2063232      229248  6297.53    0.000
Error        89        3240          36
Total        98     2066472

SOURCE       DF      SEQ SS
C42           1     2048188
C43           1         216
C44           1           1
C45           1         508
C46           1        1343
C47           1        3316
C48           1        1172
C49           1        8002
C50           1         487

Unusual Observations
Obs.    C42       C41        Fit  Stdev.Fit   Residual   St.Resid
 17    64.9   178.362    160.130      1.810     18.232      3.17R
 23    68.9   176.654    164.015      1.971     12.639      2.22R
 53    53.8   165.311    150.008      1.614     15.304      2.63R
 99    64.5   158.944    144.543      2.963     14.400      2.74R

R denotes an obs. with a large st. resid.

MTB > regress c41 4 c42 c47 c48 c50
```

```
The regression equation is
C41 = 79.1 + 1.23 C42 + 2.16 C47 - 6.04 C48 - 9.36 C50

98 cases used 2 cases contain missing values

Predictor        Coef        Stdev     t-ratio        p
Constant       79.086        4.805       16.46    0.000
C42           1.23156      0.08680       14.19    0.000
C47             2.160        1.895        1.14    0.257
C48            -6.040        1.872       -3.23    0.002
C50            -9.355        2.205       -4.24    0.000

s = 6.490       R-sq = 69.5%      R-sq(adj) = 68.2%

Analysis of Variance

SOURCE       DF          SS          MS         F        p
Regression    4      8926.2      2231.6     52.99    0.000
Error        93      3916.7        42.1
Total        97     12842.9

SOURCE       DF      SEQ SS
C42           1      7725.4
C47           1        68.7
C48           1       374.3
C50           1       757.9

Unusual Observations
Obs.     C42       C41        Fit  Stdev.Fit   Residual    St.Resid
  8     58.4   135.209    137.947      2.889     -2.738     -0.47 X
  9     54.2   130.765    132.777      2.946     -2.012     -0.35 X
 17     64.9   178.362    158.974      1.243     19.388      3.04R
 23     68.9   176.654    163.952      1.517     12.701      2.01R
 50     52.1   129.400    143.286      0.879    -13.886     -2.16R
 53     53.8   165.311    145.405      0.857     19.906      3.09R
 99     64.5   158.944    145.485      2.888     13.458      2.32RX
100     67.5   148.467    149.217      2.924     -0.749     -0.13 X

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

MTB > print c1 c2 c3 c9 c10
```

ROW	consump	temp	monday	sunday	holiday
1	*	36.7800	1	0	0
2	104.428	41.5540	0	0	0
3	103.743	35.9458	0	0	0
4	100.320	34.1945	0	0	0
5	109.285	43.9331	0	0	0
6	106.471	42.0398	0	0	0
7	93.841	42.9749	0	1	1
8	98.517	41.5569	1	0	1
9	92.245	37.9134	0	0	1
10	100.139	36.7285	0	0	0
11	113.020	40.7818	0	0	0
12	107.560	39.9786	0	0	0

13	109.513	40.8814	0	0	0
14	107.103	47.9436	0	1	0
15	106.859	43.6921	1	0	0
...					
91	85.780	28.1724	0	1	0
92	89.995	33.2663	1	0	0
93	100.446	33.5849	0	0	0
94	100.856	37.4965	0	0	0
95	123.543	46.2875	0	0	0
96	98.810	41.3563	0	0	0
97	116.906	47.7482	0	0	0
98	95.338	45.1705	0	1	1
99	121.666	46.8196	1	0	1
100	100.896	49.2046	0	0	1

```
MTB > acf c2

ACF of temp

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1   0.621                           XXXXXXXXXXXXXXXXX
  2   0.450                           XXXXXXXXXXXX
  3   0.304                           XXXXXXXXX
  4   0.213                           XXXXXX
  5   0.143                           XXXXX
  6   0.092                           XXX
  7  -0.016                           X
  8   0.094                           XXX
  9   0.055                           XX
 10  -0.023                          XX
 11   0.015                           X
 12   0.039                           XX
 13   0.069                           XXX
 14   0.020                           X
 15  -0.057                          XX
 16  -0.094                         XXX
 17  -0.009                           X
 18  -0.091                         XXX
 19  -0.088                         XXX
 20  -0.072                         XXX
```

```
MTB > pacf c2

PACF of temp

           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
  1   0.621                           XXXXXXXXXXXXXXXXX
  2   0.106                           XXXX
  3  -0.019                           X
  4   0.006                           X
  5  -0.007                           X
  6  -0.009                           X
```

```
 7  -0.125                        XXXX
 8   0.234                           XXXXXXX
 9  -0.073                         XXX
10  -0.139                        XXXX
11   0.124                           XXXX
12   0.045                           XX
13   0.026                           XX
14  -0.133                        XXXX
15  -0.026                          XX
16  -0.064                         XXX
17   0.113                           XXXX
18  -0.107                        XXXX
19  -0.009                           X
20   0.042                           XX

MTB > arima 1 0 0 c2

Estimates at each iteration
Iteration        SSE      Parameters
    0          3203.47    0.100    35.415
    1          2714.14    0.250    29.518
    2          2382.88    0.400    23.621
    3          2209.68    0.550    17.722
    4          2181.73    0.634    14.401
    5          2181.64    0.639    14.215
    6          2181.64    0.639    14.204
Relative change in each estimate less than  0.0010

Final Estimates of Parameters
Type         Estimate      St. Dev.    t-ratio
AR   1        0.6393        0.0795       8.04
Constant     14.2042        0.4720      30.09
Mean          39.378         1.309

No. of obs.:   100
Residuals:     SS =  2180.01  (backforecasts excluded)
               MS =    22.25  DF = 98

Modified Box-Pierce chisquare statistic
Lag                 12            24             36             48
Chisquare    14.6(DF=11)   37.2(DF=23)    41.0(DF=35)    49.4(DF=47)


X
```

x