NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

May 2005
3 Hours

This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.

Throughout this paper a_t represents white noise, $E(a_t) = 0$ and $E(a_t^2) = \sigma^2$.

SECTION A : Answer ALL questions from this section.

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(1,2,0) process.

[3 Marks]

2. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{l=1}^{p} \phi_\ell \rho_{k-\ell}.$$

Hence find the first 3 terms in the autocorrelation function for an AR(2) process with $\phi_1 = -0.3$ and $\phi_2 = 0.5$.

[6 Marks]

3. Derive the autocorrelation function for an ARIMA(0,0,2) process.

[4 Marks]

4. Determine if the following processes are stationary and/or invertible.

(a) $z_t - z_{t-1} = -0.3z_{t-1} + 0.1z_{t-2} + a_t - 0.4f_{t-1}$
(b) $z_t - \frac{1}{8}z_{t-1} = a_t - 0.3a_{t-1} + 2f_{t-2}$

[4 Marks]

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5. Comment briefly on the relative merits of time series methods and regression methods in producing sales forecasts.

[4 Marks]

6. Given (x_t, y_t), $t = 1, 2, \ldots, n$, describe how you would find the best linear unbiased estimates of α and β for each of the following models,

   (a) $y_t = \alpha + \beta x_t + u_t$, where $u_t = -0.5 * u_{t-1} + a_t$,

   (b) $y_t = \alpha x_t^{\beta} e^{u_t}$, where $u_t = x_t^2 a_t$.

[7 Marks]

7. Find the process followed by the residuals when an MA(1) model is fitted to an AR(1) process.

[6 Marks]

SECTION B : Answer TWO questions from this section.
Each question carries 33 marks.

8. (a) Write an ARIMA(1,0,1) model in general linear process form,

$$z_t - \mu = \sum_{j=0}^{\infty} \psi_j a_{t-j}.$$

[7 Marks]

   (b) Show that, if a stationary model is written in general linear process form, then the covariance $E((z_t - \mu)(z_{t-k} - \mu))$ can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell} \psi_{\ell} \sigma^2.$$

[5 Marks]

(c) Hence find the autocorrelation function for an ARIMA(1,0,1) process.

[7 Marks]

(d) The generalised Yule-Walker equations are

$$\rho_j = \sum_{\ell=1}^{k} \phi_{k\ell}\rho_{j-\ell} \qquad j = 1, 2, \ldots, k.$$

Use these equations to find the partial autocorrelation function, ϕ_{kk}, for an ARIMA(1,0,1) process, up to lag 3, given that $\theta_1 = -0.3$ and $\phi_1 = 0.3$.

[7 Marks]

(e) Derive the autocorelation function for an ARIMA$(0,0,1)X(0,0,1)_{12}$ process.

[8 Marks]

9. An operations research consultant has been asked to analyse daily data on the sales in thousands of dollars at a particular restaurant. The consultant used the MINITAB statistical package to produce the output given in appendix A.

Explain clearly the output given, including

(a) the reasons for using each of the commands,

(b) a summary of the conclusions of the output from each command.

(c) On the basis of the information given, specify a model which you think is most appropriate for these data. Describe any further tests you would carry out before finally recommending a model to the management.

(d) Calculate forecasts for the next two days.

[33 Marks]

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10. A student has analysed daily data on electricity consumption in a particular city.

The student used the MINITAB statistical package to produce the output given in appendix B.

(a) Explain clearly the output given, including

i. the reasons for using each of the commands,

ii. a summary of the conclusions of the output from each command.

[23 Marks]

(b) On the basis of the information given, specify a model which you think is most appropriate for these data. Use this model to calculate forecasts for the next two days, the first of which will be a holiday. Describe any further tests you would carry out before finally recommending a model to the electricity authority.

[10 Marks]

END OF EXAMINATION PAPER
