

NATIONAL UNIVERSITY OF SCIENCE AND  
TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA5253 FORECASTING

2005 Supplementary Examination  
3 Hours

**This paper contains TWO sections. Answer ALL the questions in section A and TWO questions from section B.**

Throughout this paper  $z_t$  represents an observation at time  $t$ ,  $f_t$  represents a forecast at time  $t$  and  $a_t$  represents white noise,  $E(a_t) = 0$  and  $E(a_t^2) = \sigma^2$ .

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**SECTION A : Answer ALL questions from this section.**

1. Describe the autocorrelation and partial autocorrelation functions produced by an ARIMA(2,1,1) process.

[4 Marks]

2. A series of data follows an ARIMA(2,1,1) model, with  $\phi_1 = 0.0$ ,  $\phi_2 = 0.4$ ,  $\theta_1 = -0.3$  and  $C = 4.0$ . Calculate forecasts for  $t=101$  and  $t=102$ , given that  $z_{98} = 80$ ,  $z_{99} = 90$ ,  $z_{100} = 100$  and  $f_{100} = 90$ .

[6 Marks]

3. Derive the Yule-Walker equations for an AR(p) process,

$$\rho_k = \sum_{l=1}^p \phi_l \rho_{k-l}, k = 1, 2, \dots$$

Hence find the first 3 terms in the autocorrelation function for an AR(2) process with  $\phi_1 = 0.0$  and  $\phi_2 = 0.4$ .

[8 Marks]

4. Determine if the following process is stationary and/or invertible.

$$z_t - z_{t-1} = 0.2z_{t-1} + a_t - 0.4f_{t-2}.$$

[3 Marks]

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SMA 5253

Answer ALL questions.  
 correlation and  
 process.

ARIMA(2,1,  
 casts for  $t=1$

an AR(p) process  
 $\phi_k, k=1, \dots, p$   
 relation function

or invert.  
 $f_{t-2}$

5. Given  $(x_t, y_t), t = 1, 2, \dots, n$ , describe how you would use a least squares method to find the best linear unbiased estimates of  $\alpha$  and  $\beta$  for each of the following models,
- (a)  $y_t = \alpha + \beta x_t + u_t$ , where  $u_t = x_t^2 a_t$ ,
  - (b)  $y_t = \alpha + \beta x_t + u_t$ , where  $u_t = 0.4u_{t-1} + a_t$ .

[7 Marks]

6. Derive the autocorrelation function for an ARIMA(0, 0, 1)X(0, 0, 1)<sub>4</sub> stationary process.

[6 Marks]

**SECTION B : Answer TWO questions from this section.**  
 Each question carries 33 marks.

7. (a) An ARIMA(0,0,2) process has been fitted by an ARIMA(1,0,0) model such that  $E(u_t^2)$  has been minimised, where

$$u_t = z_t - \mu - p_1(z_{t-1} - \mu).$$

- i. By finding  $p_1$  in terms of the parameters of the process, show that  $u_t$  follows an ARIMA(0,0,3) process. [4 Marks]
- ii. Derive the autocorrelation function of  $u_t$ . [4 Marks]
- iii. The generalised Yule-Walker equations are [6 Marks]

$$\rho_j = \sum_{\ell=1}^k \phi_{k\ell} \rho_{j-\ell} \quad j = 1, 2, \dots, k.$$

Use these equations to find the partial autocorrelation function,  $\phi_{kk}$ , of  $u(t)$  up to lag 3.

[6 Marks]

- iv. Find the coefficients up to lag 3 in the AR( $\infty$ ) representation of  $u(t)$ .

[5 Marks]

- (b) i. Write an  $ARIMA(1, 0, 1)X(0, 0, 1)_4$  model in general linear process form, [6 Marks]
- ii. Given that, if a stationary model is written in general linear process form, then the covariance  $E((z_t - \mu)(z_{t-k} - \mu))$  can be written as

$$\gamma_k = \sum_{\ell=0}^{\infty} \psi_{k+\ell} \psi_{\ell} \sigma^2,$$

find the autocorrelation function for an  $ARIMA(1, 0, 1)X(0, 0, 1)_4$  process.

[6 Marks]

8. An operations research consultant has been asked to analyse the weekly traffic flow over a newly constructed bridge (data in thousands of vehicles). The consultant used the MINITAB statistical package to produce the output given in appendix A.

- (a) For each MINITAB command, briefly summarize the conclusions reached from the output to that command.

[27 Marks]

- (b) Use the output to estimate traffic flow for the next three weeks.

[6 Marks]

9. A statistical consultant has been asked to analyse monthly data concerning the sales of televisions by a particular store, taking into account the varying price index.

The consultant used the MINITAB statistical package to produce the output given in appendix B.

- (a) For each MINITAB command, briefly summarize the conclusions reached from the output.

[27 Marks]

- (b) Given that the price index of the televisions is estimated to be 156 and 145, respectively, for the next two months, estimate television sales for those two months.

[6 Marks]

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END OF EXAMINATION PAPER

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SMA 5253

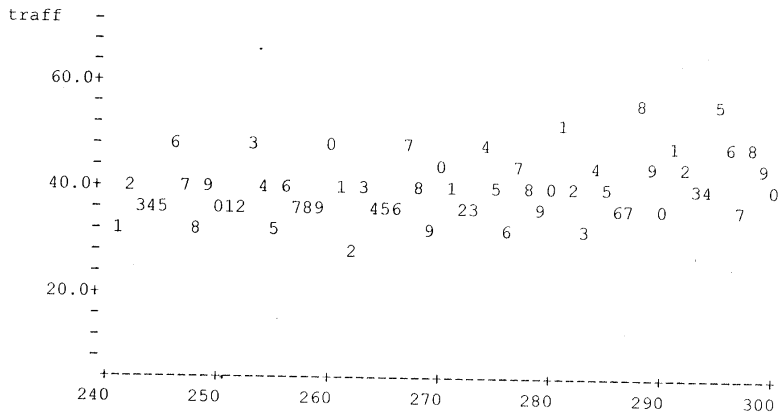
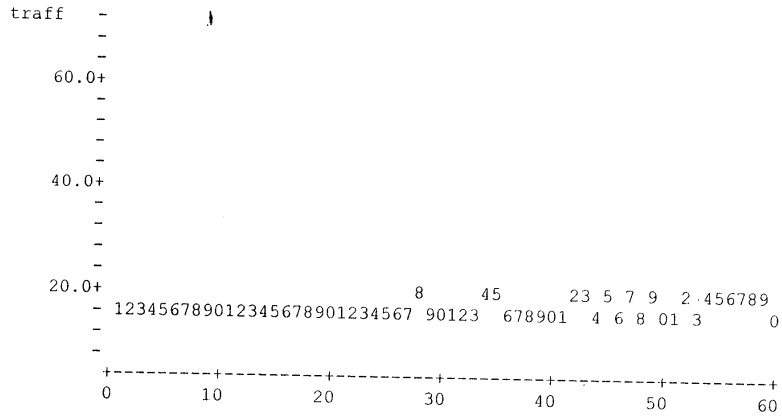
**APPENDIX A**

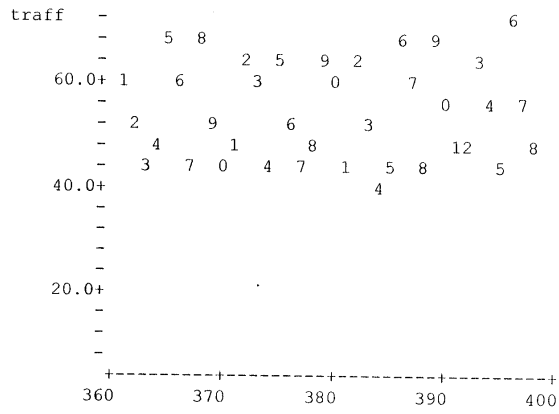
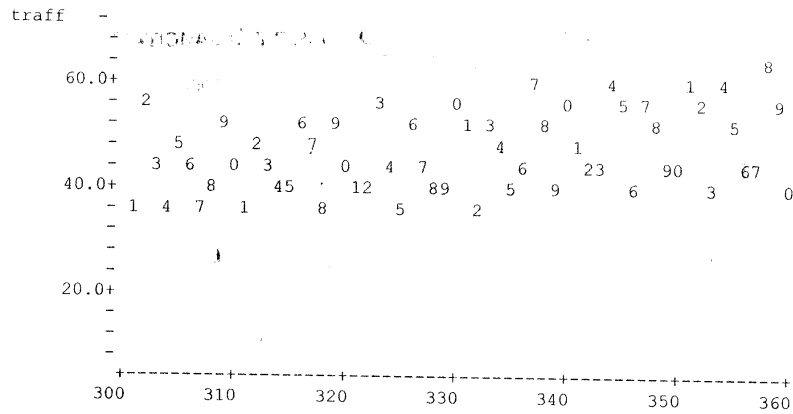
MTB > info

COLUMN	NAME	COUNT
C1	traff	398

CONSTANTS USED: NONE

MTB > tsplot c1





MTB > acf c1

ACF of traff

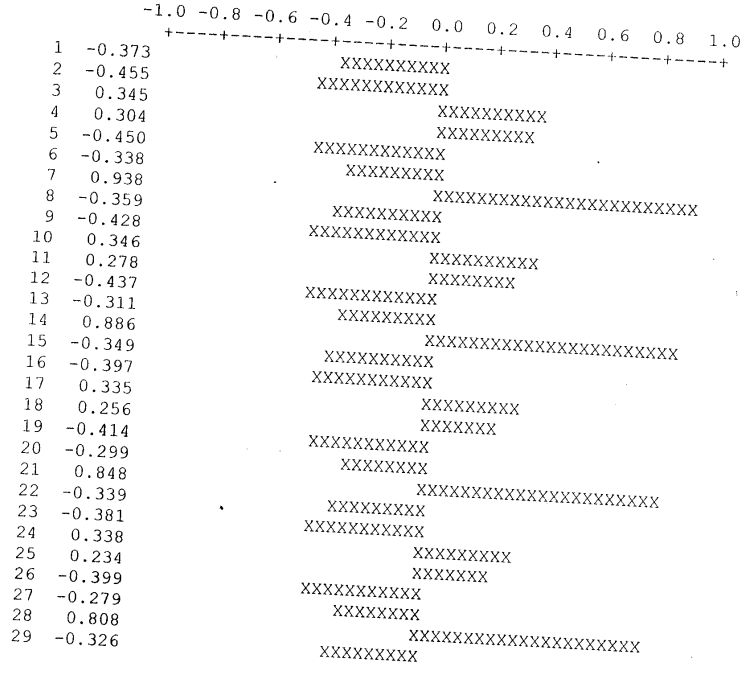
Lag	Value	Significance
1	0.841	XXXXXXXXXXXXXXXXXXXX
2	0.796	XXXXXXXXXXXXXXXXXXXX
3	0.889	XXXXXXXXXXXXXXXXXXXX
4	0.881	XXXXXXXXXXXXXXXXXXXX
5	0.775	XXXXXXXXXXXXXXXXXXXX
6	0.807	XXXXXXXXXXXXXXXXXXXX
7	0.948	XXXXXXXXXXXXXXXXXXXX
8	0.798	XXXXXXXXXXXXXXXXXXXX
9	0.758	XXXXXXXXXXXXXXXXXXXX
10	0.847	XXXXXXXXXXXXXXXXXXXX

11	0.835	
12	0.755	XXXXXXXXXXXXXXXXXXXX
13	0.765	XXXXXXXXXXXXXXXXXXXX
14	0.897	XXXXXXXXXXXXXXXXXXXX
15	0.756	XXXXXXXXXXXXXXXXXXXX
16	0.721	XXXXXXXXXXXXXXXXXXXX
17	0.807	XXXXXXXXXXXXXXXXXXXX
18	0.792	XXXXXXXXXXXXXXXXXXXX
19	0.695	XXXXXXXXXXXXXXXXXXXX
20	0.725	XXXXXXXXXXXXXXXXXXXX
21	0.851	XXXXXXXXXXXXXXXXXXXX
22	0.715	XXXXXXXXXXXXXXXXXXXX
23	0.683	XXXXXXXXXXXXXXXXXXXX
24	0.766	XXXXXXXXXXXXXXXXXXXX
25	0.748	XXXXXXXXXXXXXXXXXXXX
26	0.654	XXXXXXXXXXXXXXXXXXXX
27	0.683	XXXXXXXXXXXXXXXXXXXX
28	0.803	XXXXXXXXXXXXXXXXXXXX
29	0.672	XXXXXXXXXXXXXXXXXXXX

MTB > diff c1 c2

MTB > acf c2

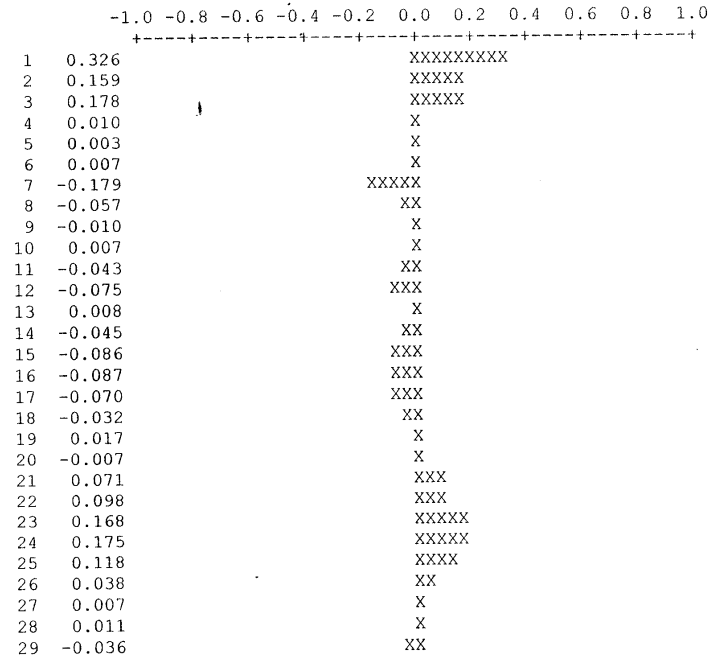
ACF of C2



MTB > diff 7 cl c3

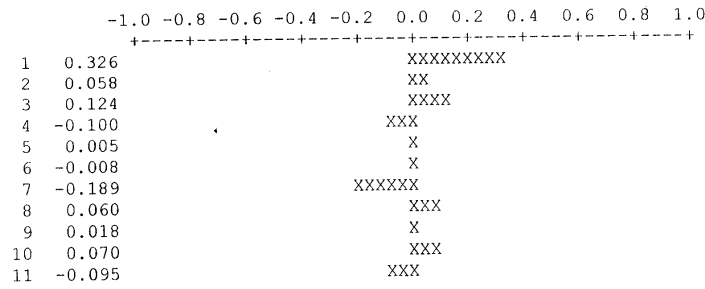
MTB > acf c3

ACF of C3



MTB > pacf c3

PACF of C3



12	-0.059	XX
13	0.068	XXX
14	-0.088	XXX
15	-0.046	XX
16	-0.065	XXX
17	0.033	XX
18	-0.005	X
19	0.012	X
20	0.009	X
21	0.074	XXX
22	0.042	XX
23	0.100	XXX
24	0.079	XXX
25	0.022	XX
26	-0.060	XXX
27	-0.047	XX
28	0.039	XX
29	-0.036	XX

MTB > arima 1 0 0 0 1 1 7 c1 c20 c30

Estimates at each iteration

Iteration	SSE	Parameters	
0	867.737	0.100	0.100
1	771.049	0.250	0.083
2	721.764	0.400	0.080
3	711.564	0.499	0.108
4	711.259	0.511	0.125
5	711.249	0.513	0.126
6	711.249	0.514	0.126
7	711.249	0.514	0.126

Relative change in each estimate less than 0.0010

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.5139	0.0450	11.42
SMA 7	0.1264	0.0529	2.39

Differencing: 0 regular, 1 seasonal of order 7

No. of obs.: Original series 398, after differencing 391

Residuals: SS = 711.183 (backforecasts excluded)  
MS = 1.828 DF = 389

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	40.2(DF=10)	52.0(DF=22)	66.9(DF=34)	77.3(DF=46)

MTB > cdf 40.2;

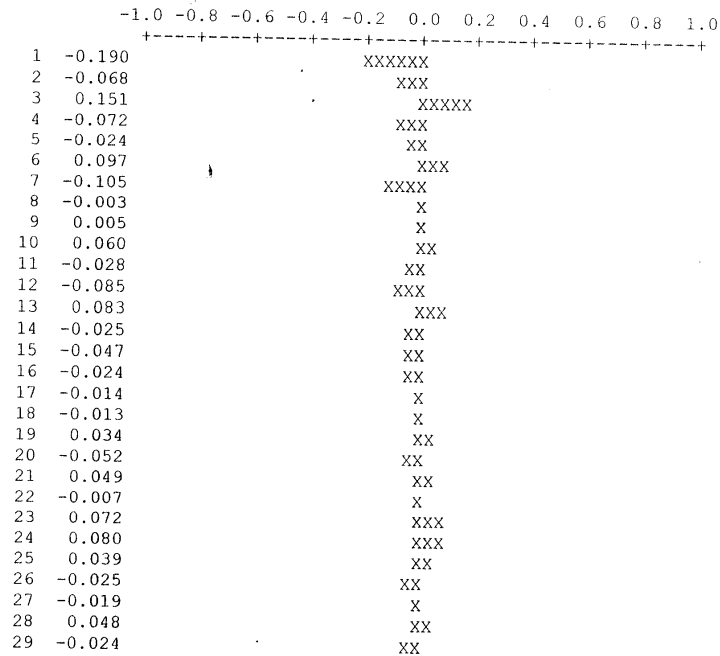
SUBC> chisq 10.

40.2000	1.0000
---------	--------



MTB > acf c20

ACF of C20



MTB > arima 1 0 1 1 1 1 7 c1 c20 c30;  
SUBC> constant.

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.6134	0.1103	5.56
SAR 7	-0.0238	0.2363	-0.10
MA 1	0.3166	0.1321	2.40
SMA 7	0.2082	0.2325	0.90
Constant	0.28354	0.03457	8.20

Differencing: 0 regular, 1 seasonal of order 7  
No. of obs.: Original series 398, after differencing 391  
Residuals: SS = 615.684 (backforecasts excluded)  
MS = 1.595 DF = 386

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	16.7 (DF= 8)	30.8 (DF=20)	44.8 (DF=32)	55.0 (DF=44)

```

MTB > cdf 16.7000
SUBC> chisq 8.
      16.7000      0.9666
MTB > let c21=c20*c20
MTB > regress c21 1 c30

```

The regression equation is  
C21 = - 1.84 + 0.104 C30

391 cases used 7 cases contain missing values

Predictor	Coef	Stdev	t-ratio	p
Constant	-1.8432	0.3813	-4.83	0.000
C30	0.10400	0.01079	9.64	0.000

s = 2.770      R-sq = 19.3%      R-sq(adj) = 19.1%

Analysis of Variance

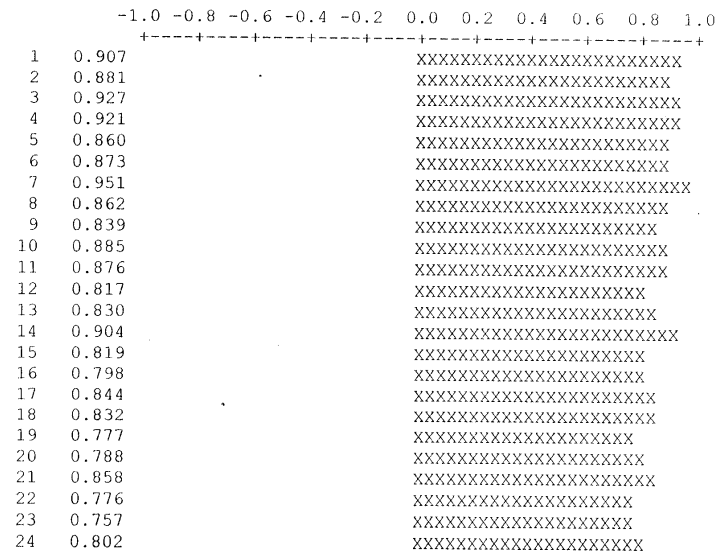
SOURCE	DF	SS	MS	F	p
Regression	1	712.76	712.76	92.86	0.000
Error	389	2985.71	7.68		
Total	390	3698.47			

```

MTB > let c10=log(c1)
MTB > acf c10

```

ACF of C10



```

25 0.788 XXXXXXXXXXXXXXXXXXXXXXXX
26 0.733 XXXXXXXXXXXXXXXXXXXXXXXX
27 0.744 XXXXXXXXXXXXXXXXXXXXXXXX
28 0.812 XXXXXXXXXXXXXXXXXXXXXXXX
29 0.733 XXXXXXXXXXXXXXXXXXXXXXXX

```

```

MTB > diff 7 c10 c11
MTB > acf c11

```

ACF of C11

```

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
          +-----+-----+-----+-----+-----+
1  0.355                                XXXXXXXXXXXXX
2  0.165                                XXXXXX
3  0.107                                XXXX
4 -0.020                                XX
5 -0.036                                XX
6 -0.085                                XXX
7 -0.277                                XXXXXXXXX
8 -0.133                                XXXX
9 -0.063                                XXX
10 -0.002                               X
11 -0.003                               X
12 -0.056                               XX
13 -0.006                               X
14 -0.030                               XX
15 -0.054                               XX
16 -0.058                               XX
17 -0.079                               XXX
18 -0.062                               XXX
19  0.041                               XX
20  0.023                               XX
21  0.064                               XXX
22  0.066                               XXX
23  0.116                               XXXX
24  0.170                               XXXXX
25  0.069                               XXX
26  0.005                               X
27 -0.036                               XX
28 -0.054                               XX
29 -0.046                               XX

```

```

MTB > pacf c11

```

PACF of C11

```

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
          +-----+-----+-----+-----+-----+
1  0.355                                XXXXXXXXXXXXX
2  0.045                                XX
3  0.041                                XX
4 -0.087                                XXX
5 -0.016                                X
6 -0.070                                XXX
7 -0.248                                XXXXXXXX

```

```

8 0.054 XX
9 0.020 X
10 0.060 XXX
11 -0.046 XX
12 -0.075 XXX
13 0.016 X
14 -0.104 XXXX
15 -0.033 XX
16 -0.036 XX
17 -0.017 X
18 -0.027 XX
19 0.054 XX
20 -0.002 X
21 0.023 XX
22 -0.001 X
23 0.075 XXX
24 0.096 XXX
25 -0.059 XX
26 -0.014 X
27 -0.047 XX
28 0.005 X
29 -0.019 X

```

```

MTB > arima 1 0 0 0 1 1 7 c10 c20 c30;
SUBC> constant.

```

```

Final Estimates of Parameters
Type      Estimate      St. Dev.  t-ratio
AR 1      0.3436      0.0477    7.21
SMA 7     0.2953      0.0490    6.02
Constant 0.014595   0.001233  11.84

```

```

Differencing: 0 regular, 1 seasonal of order 7
No. of obs.: Original series 398, after differencing 391
Residuals:  SS = 0.458926 (backforecasts excluded)
              MS = 0.001183 DF = 388

```

```

Modified Box-Pierce chisquare statistic
Lag      12      24      36      48
Chisquare 7.6(DF=10) 21.6(DF=22) 30.3(DF=34) 38.6(DF=46)

```

```

MTB > cdf 7.6;
SUBC> chisq 10.
7.6000 0.3322
MTB > cdf 21.6;
SUBC> chisq 22.
21.6000 0.5160
MTB > acf c20

```

ACF of C20

```

      -1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
      +-----+-----+-----+-----+-----+
1 -0.014 X
2 0.018 X
3 0.085 XXX
4 -0.031 XX

```

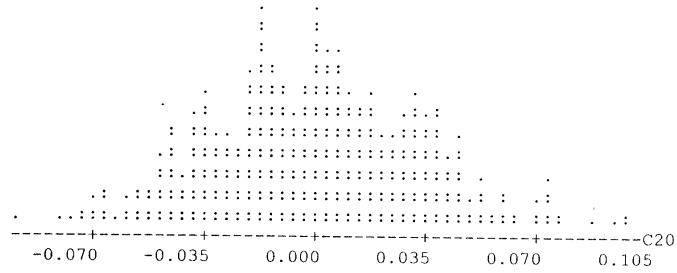
```

5 -0.016 X
6 0.017 X
7 -0.001 X
8 -0.057 XX
9 -0.025 XX
10 0.049 XX
11 -0.006 X
12 -0.059 XX
13 0.026 XX
14 -0.005 X
15 -0.052 XX
16 -0.021 XX
17 -0.006 X
18 -0.072 XXX
19 0.052 XX
20 -0.010 X
21 0.041 XX
22 -0.005 X
23 0.036 XX
24 0.137 XXXX
25 -0.025 XX
26 -0.003 X
27 -0.016 X
28 -0.014 X
29 -0.014 X

```

MTB > dotplot c20

7 POINTS MISSING OR OUT OF RANGE

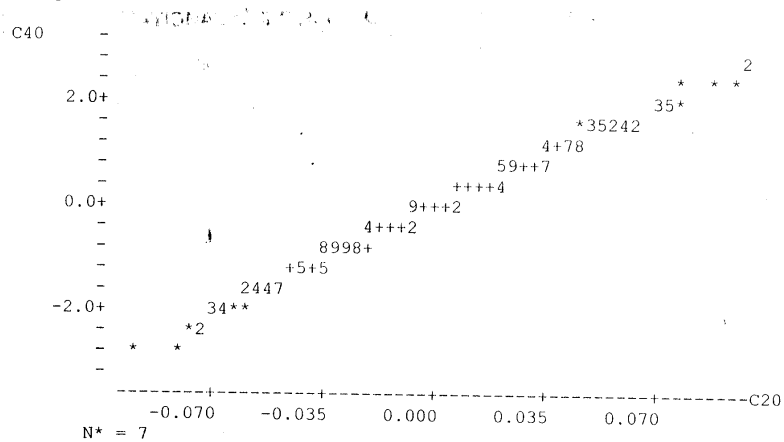


MTB > describe c20

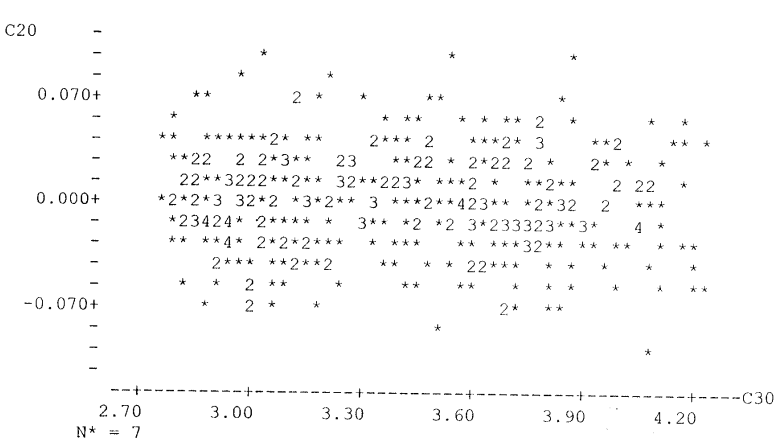
	N	N*	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
C20	391	7	0.00007	-0.00000	-0.00030	0.03430	0.00173
	MIN	MAX	Q1	Q3			
C20	-0.09383	0.09772	-0.02196	0.02411			

MTB > nscores c20 c40

MTB > plot c40 c20



MTB > plot c20 c30



MTB > print c10 c20 c30

ROW	C10	C20	C30
1	2.73925	*	*
2	2.77592	*	*
3	2.74592	*	*
4	2.78259	*	*
5	2.75259	*	*
6	2.75592	*	*
7	2.79259	*	*

8	2.76259	0.0040161	2.75857
9	2.85783	0.0592978	2.79853
10	2.82577	0.0371149	2.78866
11	2.76469	-0.0599263	2.82462
12	2.80645	0.0454149	2.76103

369	3.95863	-0.0335466	3.99217
370	3.78387	-0.0173861	3.80126
371	3.85784	0.0075252	3.85031
372	4.17260	-0.0349486	4.20755
373	4.08411	-0.0212978	4.10541
374	3.75957	0.0098732	3.74970
375	4.14792	-0.0553159	4.20324
376	3.92591	-0.0391545	3.96506
377	3.76342	-0.0289394	3.79236
378	3.86596	0.0027777	3.86318
379	4.16433	-0.0359789	4.20031
380	4.08343	-0.0187263	4.10216
381	3.75989	-0.0111258	3.77102
382	4.15368	-0.0252787	4.17896
383	3.95559	0.0015457	3.95405
384	3.72841	-0.0683545	3.79676
385	3.81829	-0.0494174	3.86771
386	4.22086	0.0476971	4.17317
387	4.07244	-0.0505411	4.12298
388	3.77334	-0.0006607	3.77400
389	4.24110	0.0607415	4.18036
390	4.01632	0.0165476	3.99977
391	3.84007	0.0560169	3.78406
392	3.86210	-0.0237489	3.88585
393	4.18091	-0.0555130	4.23642
394	4.04642	-0.0418117	4.08823
395	3.77660	-0.0025844	3.77919
396	4.27755	0.0386729	4.23888
397	4.04619	0.0076406	4.03855
398	3.85115	0.0027657	3.84839

MTB > copy c20 c21;  
SUBC> omit 1:7.  
MTB > runs 0 c21

C21

K = 0.0000

THE OBSERVED NO. OF RUNS = 197  
THE EXPECTED NO. OF RUNS = 196.4987  
195 OBSERVATIONS ABOVE K 196 BELOW  
THE TEST IS SIGNIFICANT AT 0.9595  
CANNOT REJECT AT ALPHA = 0.05

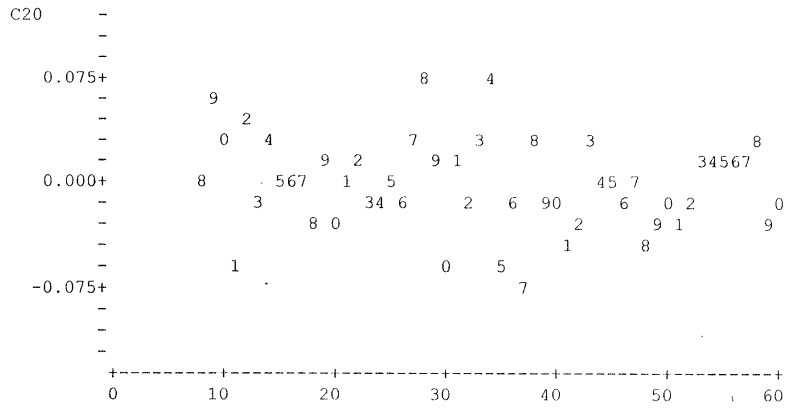
```
MTB > arima 1 0 1 1 1 1 7 c10 c50 c60;
SUBC> constant
```

```
Final Estimates of Parameters
Type      Estimate      St. Dev.  t-ratio
AR 1      0.4789      0.1275    3.76
SAR 7     -0.0296     0.1717   -0.17
MA 1      0.1537     0.1435    1.07
SMA 7     0.2752     0.1657    1.66
Constant  0.011935   0.001070  11.15
```

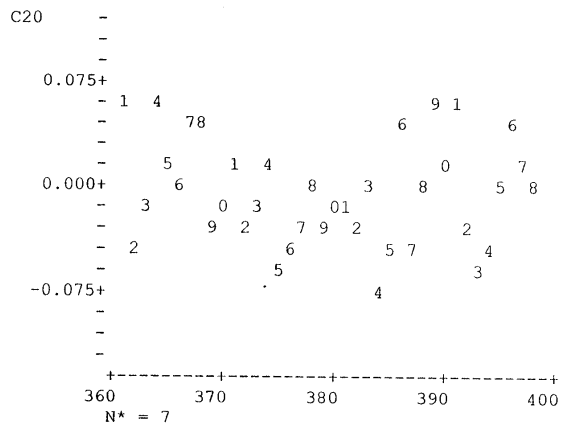
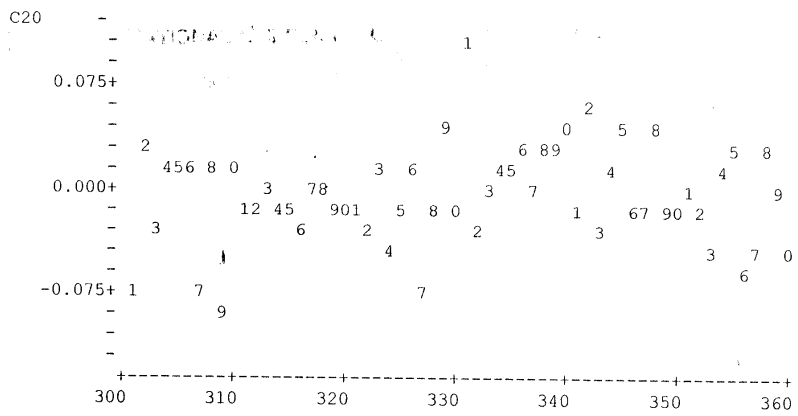
```
Differencing: 0 regular, 1 seasonal of order 7
No. of obs.: Original series 398, after differencing 391
Residuals:   SS = 0.455985 (backforecasts excluded)
              MS = 0.001181 DF = 386
```

```
Modified Box-Pierce chisquare statistic
Lag      12      24      36      48
Chisquare 7.0(DF= 8) 20.5(DF=20) 29.1(DF=32) 37.2(DF=44)
```

```
MTB > tsplot c20
```







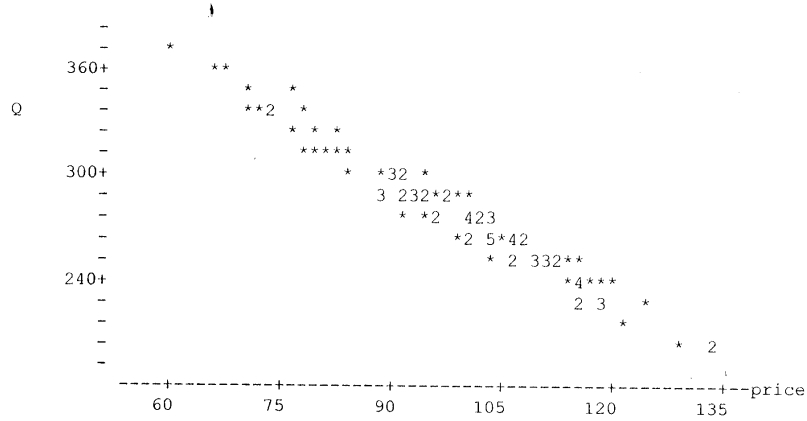
**APPENDIX B**

MTB > info

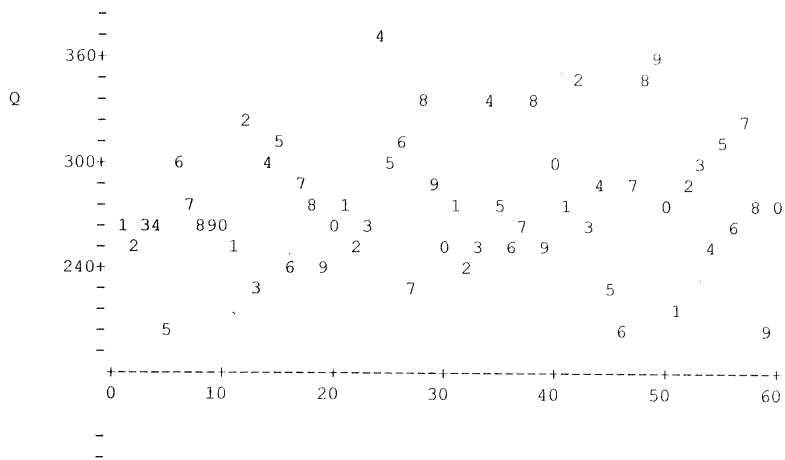
COLUMN	NAME	COUNT
C1	Q	100
C2	price	100

CONSTANTS USED: NONE

MTB > plot c1 c2

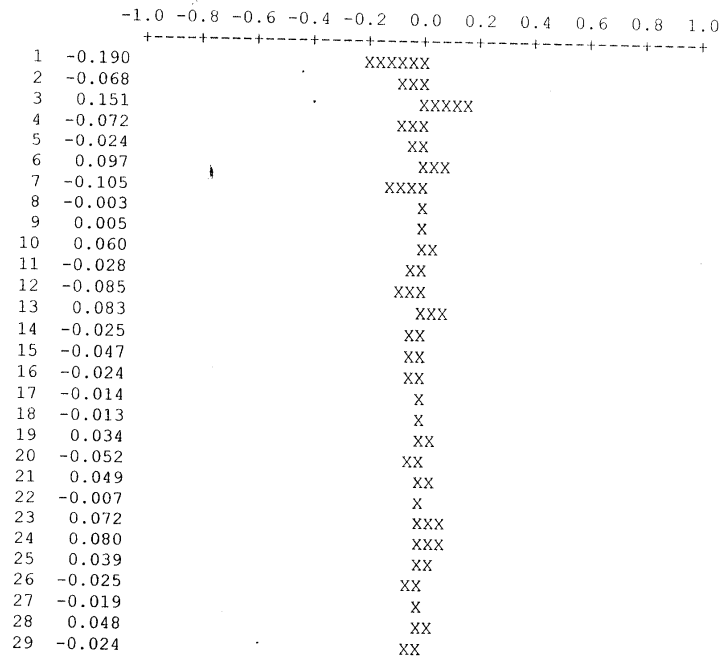


MTB > tsplot c1



MTB > acf c20

ACF of C20



MTB > arima 1 0 1 1 1 1 7 c1 c20 c30;  
SUBC> constant.

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.6134	0.1103	5.56
SAR 7	-0.0238	0.2363	-0.10
MA 1	0.3166	0.1321	2.40
SMA 7	0.2082	0.2325	0.90
Constant	0.28354	0.03457	8.20

Differencing: 0 regular, 1 seasonal of order 7  
No. of obs.: Original series 398, after differencing 391  
Residuals: SS = 615.684 (backforecasts excluded)  
MS = 1.595 DF = 386

Modified Box-Pierce chisquare statistic

Lag	12	24	36	48
Chisquare	16.7 (DF= 8)	30.8 (DF=20)	44.8 (DF=32)	55.0 (DF=44)